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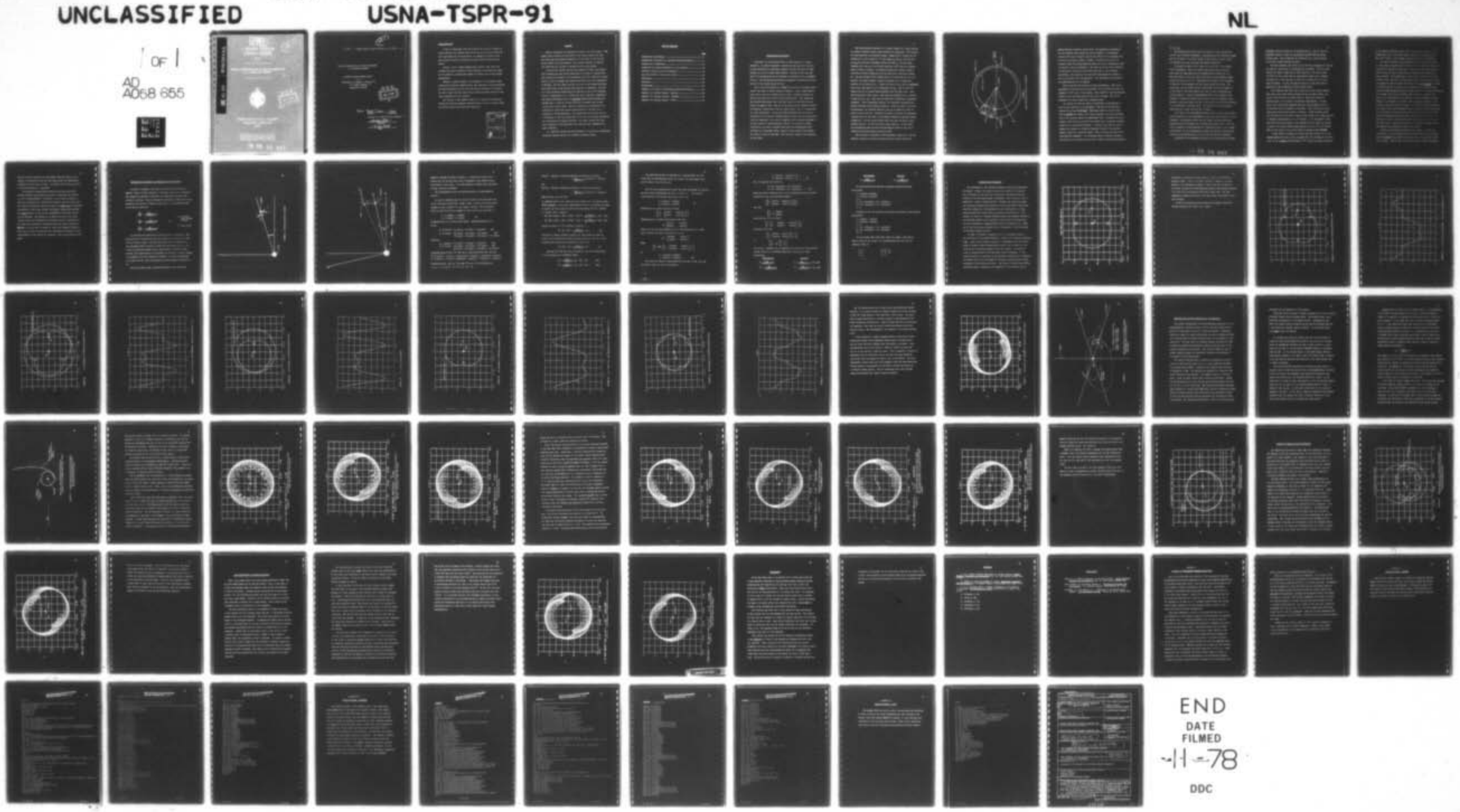

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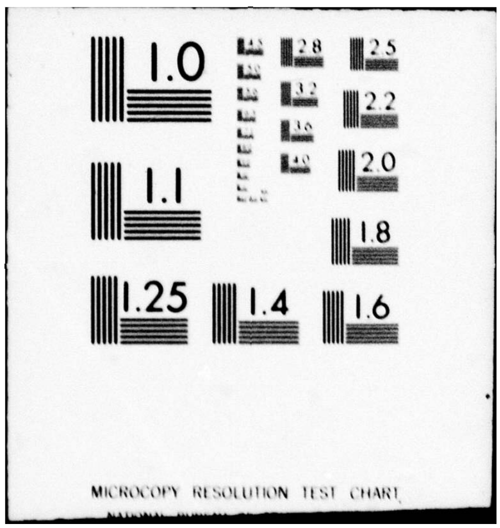
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PROJECT REPORT

NO. 91

"INITIAL CONDITIONS FOR AN ORBITAL RESONANCE IN A SATELLITE SYSTEM"

UNITED STATES NAVAL ACADEMY
ANNAPOLIS, MARYLAND
1978

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U.S.N.A. - Trident Scholar project report; no. 91 (1978)

"Initial Conditions for an Orbital Resonance in a Satellite System"

A Trident Scholar Project Report
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1 June 1978
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Acknowledgements

I wish to acknowledge first and foremost Dr. Gerald P. Calame, my project advisor, who inspired this report and who was always available with assistance over the rough spots. He managed to keep me on the right track throughout the year and provided valuable insights when needed.

Second, I wish to thank Captain Robert Kimble, USMC, whose APL programs were used to generate visual displayed for filming the escape on the computer; a process that became an integral part of the project presentation.

Thirdly, a special thanks to the personnel of the Computer Aided Design and Interactive Graphics group, Paco Rodriguez, Steve Satterfield, and Doug Richardson, for the help they gave me throughout the project. The use of their frame by frame filming techniques added greatly to the final presentation of the project.

And finally, a very special thanks to the Dartmouth Time-Sharing System and the entire Academic Computing Center without the use of which the entire project would have been impossible.

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Abstract

Orbital resonances are widespread throughout the solar system. They appear mainly between the satellites of the Jovian planets, but also are found among the planets themselves as in the case of the 3/2 resonance between Pluto and Neptune. This project undertook to examine the conditions out of which a resonance could be established.

Of particular interest was the 3/2 resonance of Pluto and Neptune. This resonance has been firmly established by a number of researchers. This project also relates to the hypothesis that Pluto might have originated as a moon of Neptune that escaped and entered an orbit independent of Neptune. The last major objection to this is that Neptune and Pluto never come close to one another since they are in resonance. The lack of a close encounter between Pluto and Neptune is due to the resonance and not a consequence of it. Through the use of numerical integration this paper establishes that it is <u>possible</u> that if Pluto escaped from Neptune then it could go into an orbit and a resonance very similar to that which exists today. Once that resonance is established it is impossible to regain the initial conditions due to the fact that a resonance is self-perpetuating. The integrations were carried out for a time equal to over one million years in a model solar system containing all the planets from Jupiter outwards. This paper removes that last major objection to the hypothesis that Pluto may have originated as a moon of Neptune.

An additional result was the development of a system for identifying resonances through the use of a rotating coordinate system.

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Introduction and Proposal

Throughout the solar system there are many examples of orbital resonance. An orbital resonance is defined as any system in which two or more satellites are orbiting a primary such that the ratio of their periods is a ratio of small whole numbers. Periods of this type are said to be commensurate and the resonance is referred to as a commensurability. When discussing satellites and primaries included also are the motions of the planets around the Sun.

One of the best examples of a simple two satellite resonance exists between two moons of Saturn, Titan and Hyperion. Here a large massive moon, Titan, moves in a nearly circular orbit. Hyperion, however, is smaller and moves in an orbit of large eccentricity. This system has been studied extensively and it has been found that the ratio of the periods is equal to 4/3. That is, if n_1 and n_2 are the periods of Titan and Hyperion, respectively, then the relation $4n_1 - 3n_2 = 0$ is satisfied to a very high degree. It has also been determined that the point of conjunction or the point of closest approach between the moons oscillates about the apocenter of the orbit of Hyperion with a maximum amplitude of 36° and a period of approxiamately 2 years.[1] In addition to these oscillations the line of apsides about which the oscillations occur precesses in a retrograde sense, opposite to the motion of the bodies, at a rate of about 19° per year. This is also a result of the resonance of the orbits.

The Titan-Hyperion resonance is a classic example of a large category of orbital resonances termed large eccentricity resonances. The mechanism for this resonance is relatively simple. Suppose that a primary has two satellites in orbit around it. If the moons, m_1 and m_2, are such that $m_1 \gg m_2$ and they are in coplanar orbits such that m_1 is in an inner circular orbit and m_2 is in an outer eccentric orbit then the situation shown in Figure 1 exists. If initially conjunction occurs at point A then shortly before conjunction the two moons are in positions labeled #1. The gravitational force between them can be resolved into two components, one normal inwards towards the primary and one tangential, opposite to the motion of m_2. Similar forces act on m_1 but are negligible due to the large mass of m_1 in comparison with m_2. The net effect of these forces is to remove angular momentum from the small moon. Once past conjunction there exists a time when the satellites are in position #2 such that $\theta_1 = \theta_2$. Here also the force has a normal and a tangential component. After conjunction, however, the tangential component adds angular momentum to the smaller satellite. Since, at point A the orbits are diverging, the distance between m_1 and m_2 is greater at point #2 than it was at point #1. Therefore the force is less and the tangential component does not add back as much angular momentum as was lost before conjunction. The net overall effect after one cycle is that m_2 loses angular momentum. To stay in orbit it must speed up. If the orbits are commensurate then the next time conjunction occurs it will be at some point slightly advanced along the orbit.

The situation is reversed if the conjunction occurs at B. Now the additive tangential component dominates and the smaller moon gains

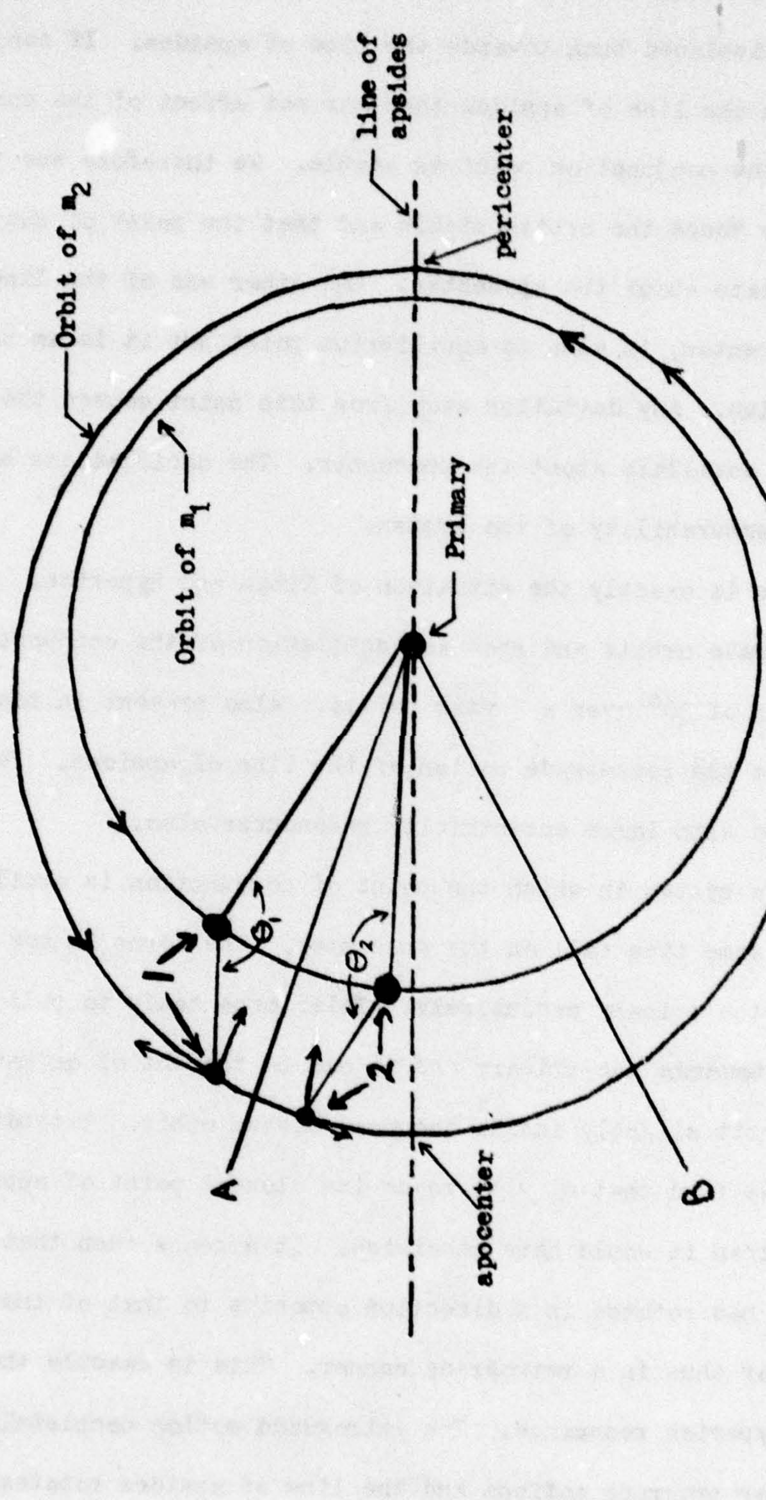

FIGURE 1.
Large Eccentricity Stability Mechanism

angular momentum therefore slowing down. The successive conjunctions will be displaced back towards the line of apsides. If conjunction occurs on the line of apsides then the net effect of the components cancel and the conjunction point is stable. We therefore see that the resonance keeps the orbits stable and that the point of conjunction tends to oscillate about the apocenter. The other end of the line of apsides, the pericenter, is also an equilibrium point but it is an unstable equilibrium. Any deviation away from this point causes the system to begin to oscillate about the apocenter. The oscillations help preserve the commensurability of the system.

This is exactly the situation of Titan and Hyperion. They are in commensurate orbits and show an oscillation of the conjunction about the apocenter of 36° over a 2 year period. Also present in the Titan-Hyperion system is the retrograde motion of the line of apsides. This can be explained with large eccentricity resonances also.

In a system in which the point of conjunction is oscillating it will at some time fall on the apocenter. The force is now directed inwards the primary ~~exclusively~~. This force tends to pull the outer moon in ~~towards~~ the primary and it can be thought of as having traveled on an orbit slightly inside the unperturbed orbit. Extending this orbit around we find that m_2 will reach its closest point of approach slightly sooner than it would have otherwise. It appears then that the line of apsides has rotated in a direction opposite to that of the satellite motion or thus in a retrograde manner. This is exactly the case in the Titan-Hyperion resonance. The retrograde motion completely dominates any other prograde motions and the line of apsides rotates approxiamately

19^o per year.

The Titan-Hyperion case is only one example of many systems that exhibit these resonance phenomenon. Resonances have been detected between most of the moons of the outer planet, in the rings of Saturn, within the asteroid belt, and even among the planets themselves.

Of primary interest to this report are the orbits of the planets Pluto and Neptune. There are a great many facets about the orbits which point out that some kind of resonance exists between the two. The first factor is that the orbital periods are approxiamately commensurate in a ratio of 3 to 2. This leads us to suspect a 3/2 resonance. In addition, this resonance is of the large eccentricity type. The system has all the correct initial conditions; the mass of Neptune is on the order of 100 times greater than that of Pluto, satisfying the $m_1 \gg m_2$ condition. Pluto's orbital eccentricity is the largest of the planets; so large that Pluto's orbit crosses inside of Neptune's orbit. Neptune, on the other hand, has one of the smallest eccentricities putting it in essentially a circular orbit. The system has all the necessary conditions for a large eccentricity resonance.

If we look at the Pluto-Neptune system today we see the same effects that are present in the Titan-Hyperion system that characterize an orbital resonance. Conjunctions between the two planets oscillate about the apocenter with the apocenter as the stable equilibrium point. Also the line of apsides of Pluto precesses in a retrograde motion. All of these similarities with the Titan-Hyperion system have led us to believe that today Pluto and Neptune exist in a 3/2 large eccentricity resonance.

In addition to the features mentioned above, Pluto exhibits a third

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phenomenon possibly caused by the commensurability. This is that the resonance appears to keep the conjunctions near 90° from the mutual orbital nodes. This eliminates the possibility of a close encounter or collision between Neptune and Pluto.

The major question that arises is: how did this resonance get started? One theory is that Pluto is an escaped moon of Neptune. There are many factors that seem to make this at least possible. The first is that the orbit of Pluto comes close to and actually inside that of Neptune. A second fact is that the mass of Pluto is low enough for it to be a moon of Neptune. In fact, the mass of Pluto is of the same order of magnitude as one of Neptune's current moons, Triton.

If we theorize that Pluto was at one time in orbit around Neptune then, due to the tidal effects between the two, we expect Pluto's rotation to have slowed to the point where it would have a period of rotation equal to its period of revolution about Neptune. In other words, Pluto would always keep the same side towards Neptune much as our own Moon does with respect to the Earth. After ejection from orbit Pluto might keep the same period of rotation. Indeed measured values for the period are very close to what we would expect for a large satellite of Neptune, approxiamately 5.5×10^{5} seconds. If we use Kepler's Third Law which relates the period to the distance from the primary to compute a value for an orbital radius around Neptune we obtain 3.8×10^{8} meters. This is very close to the orbital radius of Neptune's moon, Triton.

A third piece of evidence that Pluto might have evolved out of the Neptune system of moons is that it appears that the inclination of Pluto's orbit to the ecliptic (approxiamately 17°) is about the same inclination

as the orbits of Neptune's moons. Pluto's large inclination, as contrasted with that of the other planets which have inclinations of at most 3.5° (Venus - 3.39°), makes this seem more than a coincidence.

The last and possibly most conclusive piece of evidence in favor of Pluto being an escaped moon of Neptune is the irregular structure of the orbits of Neptune's present moons. Neptune has two moons: Triton, which is slightly smaller than Pluto, and Nereid, which resembles a stray asteroid. The irregularity is that Triton, although in a nearly circular orbit, revolves in a retrograde manner. This in itself is not too unusual but then we notice that Triton is the only massive satellite in the solar system to exhibit this retrograde motion: all the other satellites with retrograde motion are small enough to be captured asteroids. If we assume that Triton and Neptune were created at the same time, which is not unreasonable, then something must have happened to cause Triton to turn around in its orbit. That something is thought to be a close or near collision with Pluto.[2] If at one time Pluto and Triton were both in orbit around Neptune then the situation exists whereby the two could have an interaction which boosted Pluto's speed so it could escape from Neptune and go into orbit around the Sun on its own. A corollary to this is the fact that Pluto resembles a moon, small and icy, more than it resembles the gas giants of Jupiter, Saturn, Uranus, and Neptune which populate the outer solar system.

However, one difficulty results in this theory. If Pluto did originate as a moon of Neptune we would expect that Pluto and Neptune would ocasionally come close to each other due to the cyclical nature of the orbits. They do not! In fact the orbits of Pluto and Neptune

never do actually intersect and the planets never get close to one another, an observation which has been used against the Neptune-moon hypothesis for the origin of Pluto. The reason behind this may be due to the establishment of a resonance.

The resonance between Neptune and Pluto has been established through numerical integrations and orbital observations to be stable in a 3/2 commensurability.[3] Therefore, if the present situation is taken and integrated both backward and forward in time the orbits will not change significantly because the system is <u>locked into</u> the resonance. The problem is that, under resonant conditions, integrating backwards will <u>never</u> bring us to the initial conditions of the escape that set up the resonance. The analogy may be made to the case of an object falling through the atmosphere with terminal velocity: observation of the final state provides no information about the intial conditions. However if we start with the moment of escape and integrate forward in time, then it might be determined what conditions are necessary for the establishment of the present 3/2 resonance. Such is the intent of this paper.

Mathematical Conversion to a Rotating Coordinate System

In order to integrate, over time, the position and velocity of spatial bodies, we must first have a coordinate system for a reference. The simplest and easiest to work with is a Cartesian reference frame centered on the Sun. This is illustrated in Figure 2. The point (x,y,z) represents the position of Pluto at some time t. The acceleration due to the Sun is written as:

$$\frac{d^2x}{dt^2} = -\frac{GMx}{(x^2+y^2+z^2)^{3/2}}$$

$$\frac{d^2y}{dt^2} = -\frac{GMy}{(x^2+y^2+z^2)^{3/2}} \qquad (1.)$$

$$\frac{d^2z}{dt^2} = -\frac{GMz}{(x^2+y^2+z^2)^{3/2}}$$

G - Universal Gravitational constant

M - Mass of Sun

In this form the equations are relatively easy to work with. Perturbations due to Neptune and the other planets are added with the addition of similar terms of the same form except that the distances of the perturbing planet relative to Pluto are substituted for x, y, and z and the perturbing planets mass is substituted for the Sun's mass. However, the interpretation of the results is difficult if we are looking for changes in one orbit relative to another. It would be beneficial if we could fix one planet and observe only Pluto's motion relative to it.

This can be done using a rotating coordinate frame fixed with

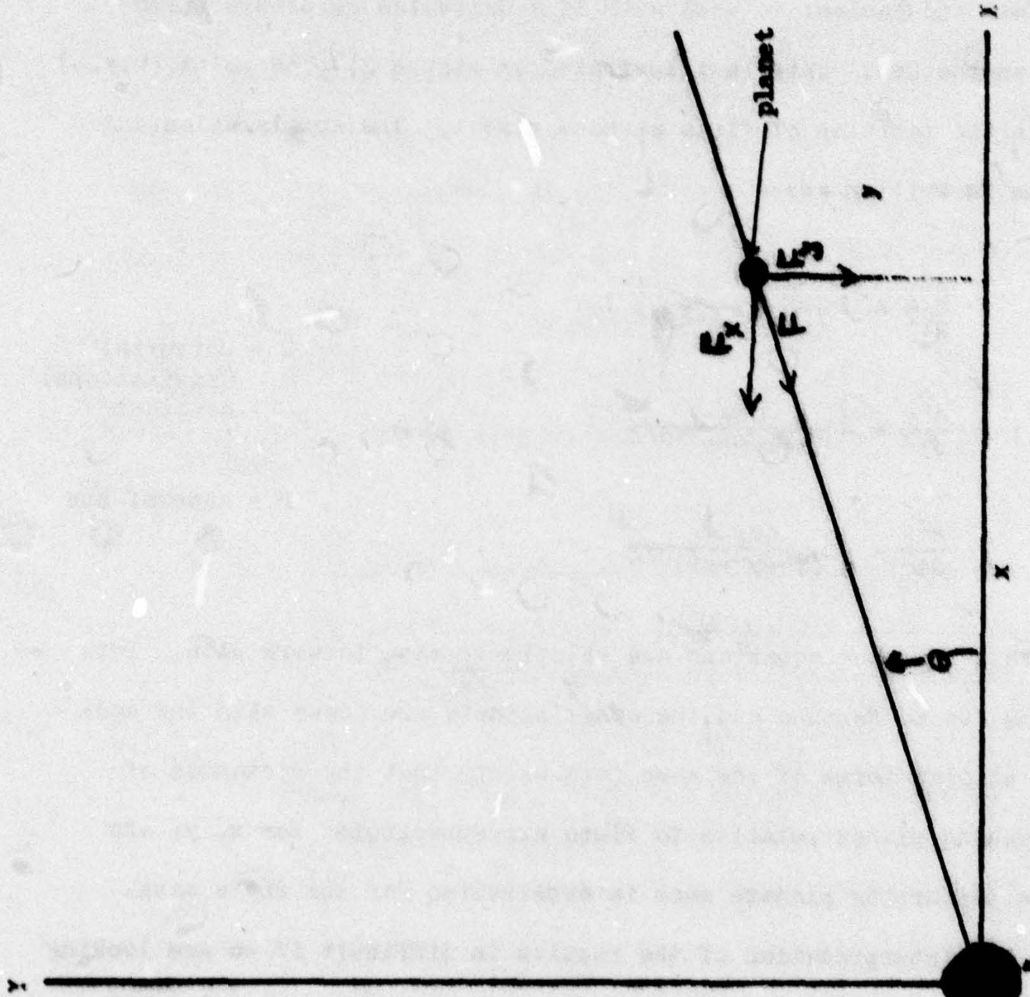


FIGURE 2. Cartesian Coordinate System

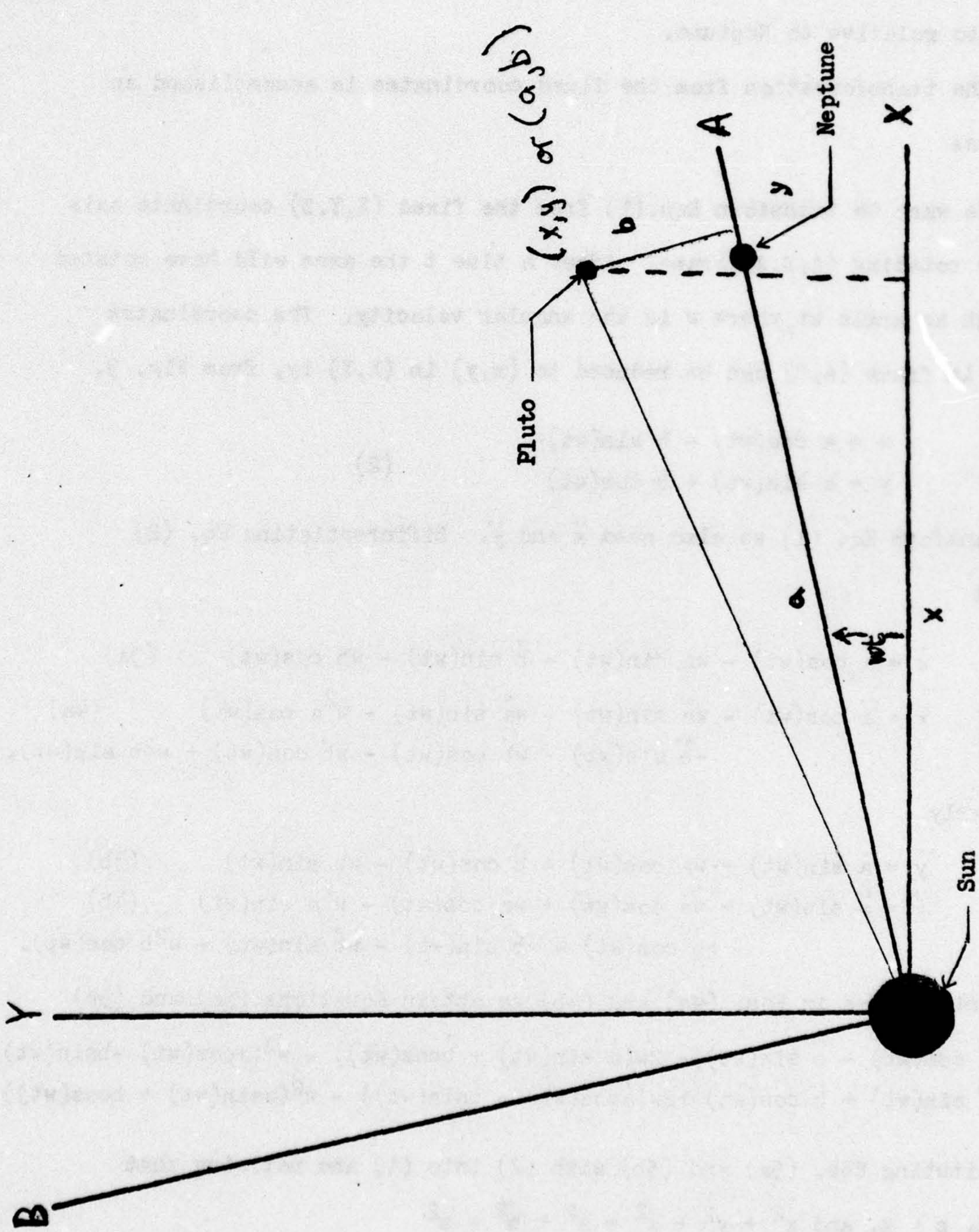


FIGURE 3. Rotating Coordinate System with Pluto and Neptune

respect to Neptune as shown in Figure 3. A second set of axes (A,B) rotates with the angular speed equal to Neptune's; thus Neptune always stays fixed on the x-axis. It is now possible to observe only the motion of Pluto relative to Neptune.

The transformation from the fixed coordinates is accomplished as follows:

We want to transform Eqs.(1) from the fixed (X,Y,Z) coordinate axis to the rotating (A,B,Z) frame. After a time t the axes will have rotated through an angle wt, where w is the angular velocity. The coordinates (a,b) in frame (A,B) can be reduced to (x,y) in (X,Y) by, from Fig. 3.

$$\begin{aligned} x &= a\cos(wt) - b\sin(wt) \\ y &= a\sin(wt) + b\cos(wt) \end{aligned} \qquad (2)$$

To transform Eq. (1) we also need $\ddot{x}$ and $\ddot{y}$. Differentiating Eq. (2) we get

$$\dot{x} = \dot{a}\cos(wt) - wa\sin(wt) - \dot{b}\sin(wt) - wb\cos(wt) \qquad (3a)$$

$$\begin{aligned} \ddot{x} = \ddot{a}\cos(wt) - w\dot{a}\sin(wt) - w\dot{a}\sin(wt) - w^2 a\cos(wt) \qquad (4a) \\ -\ddot{b}\sin(wt) - w\dot{b}\cos(wt) - w\dot{b}\cos(wt) + w^2 b\sin(wt). \end{aligned}$$

Similarly

$$\dot{y} = \dot{a}\sin(wt) + wa\cos(wt) + \dot{b}\cos(wt) - wb\sin(wt) \qquad (3b)$$

$$\begin{aligned} \ddot{y} = \ddot{a}\sin(wt) + w\dot{a}\cos(wt) + w\dot{a}\cos(wt) - w^2 a\sin(wt) \qquad (4b) \\ +\ddot{b}\cos(wt) - w\dot{b}\sin(wt) - w\dot{b}\sin(wt) - w^2 b\cos(wt). \end{aligned}$$

Combining terms in Eqs. (4a) and (4b) we obtain Equations (5a) and (5b)

$$\ddot{x} = \ddot{a}\cos(wt) - \ddot{b}\sin(wt) - 2w(\dot{a}\sin(wt) + \dot{b}\cos(wt)) - w^2(a\cos(wt) - b\sin(wt))$$

$$\ddot{y} = \ddot{a}\sin(wt) + \ddot{b}\cos(wt) + 2w(\dot{a}\cos(wt) - \dot{b}\sin(wt)) - w^2(a\sin(wt) + b\cos(wt))$$

Substituting Eqs. (5a) and (5b) with (2) into (1) and noticing that

$z = z$, $z = z$, and $x^2 + y^2 + z^2 = a^2 + b^2 + z^2$

$$\ddot{a}\cos(wt) - \ddot{b}\sin(wt) - 2w(\dot{a}\sin(wt) + \dot{b}\cos(wt)) - w^2(a\cos(wt) - b\sin(wt))$$
$$= -\frac{GM}{(a^2+b^2+z^2)^{3/2}} (a\cos(wt) - b\sin(wt))$$

and

$$\ddot{a}\sin(wt) + \ddot{b}\cos(wt) + 2w(\dot{a}\cos(wt) - \dot{b}\sin(wt)) - w^2(a\sin(wt) + b\cos(wt))$$
$$= -\frac{GM}{(a^2+b^2+z^2)^{3/2}} (a\sin(wt) + b\sin(wt)).$$

These are Eqs. (6a) and (6b).

To simplify these we will leave out the z term as the z coordinate transforms unchanged. We now use a mathematical trick of multiplying the first by cos(wt) and the second by sin(wt) and then adding the two. Letting $s = \sin(wt)$ and $c = \cos(wt)$:

$$\ddot{a}c^2 - \ddot{b}sc - 2w(\dot{a}sc + \dot{b}c^2) - w^2(ac^2 - bsc) = -\frac{GM}{(a^2+b^2)^{3/2}} (ac^2 - bsc)$$
$$\ddot{a}s^2 + \ddot{b}sc + 2w(\dot{a}sc - \dot{b}s^2) - w^2(as^2 + bsc) = -\frac{GM}{(a^2+b^2)^{3/2}} (as^2 + bsc)$$

Adding and using $s^2 + c^2 = \sin^2(wt) + \cos^2(wt) = 1$

$$\ddot{a} - 2w\dot{b} - w^2 a = -\frac{GM}{(a^2+b^2)^{3/2}} (a) \;. \qquad (7a)$$

Performing a similar operation again; only multiplying the first by sin(wt) and the second by cos(wt) and then subtracting the two. After expanding and collecting all terms we are left with:

$$\ddot{b} + 2w\dot{a} - w^2 b = -\frac{GM}{(a^2+b^2)^{3/2}} (b). \qquad (7b)$$

Equations (7a) and (7b) can be written to give the accelerations in the rotating (A,B) coordinate frame:

$$\ddot{a} = -\frac{GM}{(a^2+b^2)^{3/2}} (a) + w^2 a + 2w\dot{b} \qquad (8a)$$
$$\ddot{b} = -\frac{GM}{(a^2+b^2)^{3/2}} (b) + w^2 b - 2w\dot{a} \;. \qquad (8b)$$

The equations are what is expected in a rotating frame with the first term the gravitational force, the second, the centrifugal force, and the third, the coriolis term.

Now that the accelerations in (A,B) have been determined the position and the velocity transformations must be worked out. The position transformations have already been given in Eq. (2):

$$x = a\cos(wt) - b\sin(wt)$$
$$y = a\sin(wt) + b\cos(wt) \quad . \qquad (2)$$

Writing this as a matrix equation we have:

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \cos(wt) & -\sin(wt) \\ \sin(wt) & \cos(wt) \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix}$$

Defining $\underline{T}$ as the transformation matrix such that

$$\underline{T} = \begin{bmatrix} \cos(wt) & -\sin(wt) \\ \sin(wt) & \cos(wt) \end{bmatrix}$$

Then to go from the non-rotating (X,Y) to the rotating (A,B), we need only to invert the matrix and apply it to (a,b)

$$\underline{T}^{-1} = \begin{bmatrix} \cos(wt) & \sin(wt) \\ -\sin(wt) & \cos(wt) \end{bmatrix}$$

Thus,

$$\begin{bmatrix} a \\ b \end{bmatrix} = \underline{T}^{-1} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \cos(wt) & \sin(wt) \\ -\sin(wt) & \cos(wt) \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

or

$$a = x\cos(wt) + y\sin(wt)$$
$$b = -x\sin(wt) + y\cos(wt) \quad . \qquad (9)$$

To obtain the velocity transformations we go back to Eqs. (3a) and (3b) which result in, after rearrangeing:

$$\dot{x} = \dot{a}\cos(wt) - \dot{b}\sin(wt) - wy$$
$$\dot{y} = \dot{a}\sin(wt) + \dot{b}\cos(wt) + wx \quad . \qquad (10)$$

Eqs. (3) can also be written as:

$$\dot{x} = (\dot{a} - wb)\cos(wt) - (\dot{b} + wa)\sin(wt)$$
$$\dot{y} = (\dot{a} - wb)\sin(wt) + (\dot{b} + wa)\cos(wt) \quad . \qquad (11)$$

These two sets of equations give us the transformation matrix between velocities in the two coordinate systems:

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} \cos(wt) & -\sin(wt) \\ \sin(wt) & \cos(wt) \end{bmatrix} \begin{bmatrix} \dot{a}-wb \\ \dot{b}+wa \end{bmatrix}$$

and thus,

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \underline{T} \begin{bmatrix} \dot{a}-wb \\ \dot{b}+wa \end{bmatrix}$$

Rewritting Eq. (10), we obtain

$$\begin{bmatrix} \dot{x} + wy \\ \dot{y} - wx \end{bmatrix} = \begin{bmatrix} \cos(wt) & -\sin(wt) \\ \sin(wt) & \cos(wt) \end{bmatrix} \begin{bmatrix} \dot{a} \\ \dot{b} \end{bmatrix}$$

Inverting this gives:

$$\begin{bmatrix} \dot{a} \\ \dot{b} \end{bmatrix} = \begin{bmatrix} \cos(wt) & \sin(wt) \\ -\sin(wt) & \cos(wt) \end{bmatrix} \begin{bmatrix} \dot{x} + wy \\ \dot{y} - wx \end{bmatrix}$$

or

$$\begin{bmatrix} \dot{a} \\ \dot{b} \end{bmatrix} = \underline{T}^{-1} \begin{bmatrix} \dot{x} + wy \\ \dot{y} - wx \end{bmatrix}$$

Now we have a complete set of equations and coordinate transformations between fixed (X,Y) coordinate frame and rotating (A,B) frame.

Summarizing:

<u>Non-rotating</u>	<u>Rotating</u>
$\ddot{x} = \dfrac{-GMx}{(x^2+y^2+z^2)^{3/2}}$	$\ddot{a} = -\dfrac{GMa}{(a^2+b^2+z^2)^{3/2}} + w^2a + 2w\dot{b}$
$\ddot{y} = -\dfrac{GMy}{(x^2+y^2+z^2)^{3/2}}$	$\ddot{b} = -\dfrac{GMb}{(a^2+b^2+z^2)^{3/2}} + w^2a - 2w\dot{a}$

Non-rotating	Rotating
$\ddot{z} = \frac{-GMz}{(x^2+y^2+z^2)^{3/2}}$	$\ddot{z} = \frac{-GMz}{(a^2+b^2+z^2)^{3/2}}$

To transform from non-rotating to rotaing coordinates knowing $x, y, z, \dot{x}, \dot{y}, \dot{z}$:

$$a = x\cos(wt) + y\sin(wt)$$
$$b = -x\sin(wt) + y\cos(wt)$$
$$z = z$$
$$\dot{a} = (\dot{x} + wy)\cos(wt) + (\dot{y} - wx)\sin(wt)$$
$$\dot{b} = -(\dot{x} + wy)\sin(wt) + (\dot{y} - wx)\cos(wt)$$
$$\dot{z} = \dot{z}$$

Similarly to go from rotating to non-rotating reference frames knowing $a, b, z, \dot{a}, \dot{b}, \dot{z}$:

$$x = a\cos(wt) - b\sin(wt)$$
$$y = a\sin(wt) + b\cos(wt)$$
$$z = z$$
$$\dot{x} = (\dot{a} - wb)\cos(wt) - (\dot{b} + wa)\sin(wt)$$
$$\dot{y} = (\dot{a} - wb)\sin(wt) + (\dot{b} + wa)\cos(wt)$$
$$\dot{z} = \dot{z}$$

In the extreme case where wt=0, which will appear later when we wish to start on the x-axis, the transformations from the fixed to rotating reduce to:

$a = x$	$\dot{a} = \dot{x} + wy$
$b = y$	$\dot{b} = \dot{y} - wx$
$z = z$	$\dot{z} = \dot{z}$

Recognition of Resonances

By transforming to the rotating coordinate system three advantages are gained. Firstly, the motion of one of the planets is eliminated from the picture so that now only the motion of the perturbed planet relative to the perturbing planet is observed. This is exactly what is needed for a study of resonant motion. Secondly, by fixing one planet the calculations become more accurate over long time periods because any numerical integration of perturbations is now referenced to a fixed non-calculated point rather than to a moving calculated point. And thirdly, by going to a rotaing coordinate system the graphical display of the perturbed planet's motion provides an easy method for determining the exact resonance or commensurability that the planets are in. Thus, one of the first results obtained was the development of this technique for identying resonant orbits.

In order to identify a resonant orbit it is necessary merely to plot the motion of one planet relative to another in a rotating coordinate frame. Such a plot is shown in Figure 4. Remembering that the interior planet is always fixed on the positive x-axis it can be shown that the loops are the perihelion passage of the outer planet. The number of loops is characteristic of the particular orbital resonance. If the resonance type (i.e. the ratio of the periods) is expressed as a fraction, for example 3/2 or 4/3, the number of loops per complete pattern is the denominator of the fraction or ratio. The relative orbital period of the perturbing planet, indicated by the numerator of the fraction can be

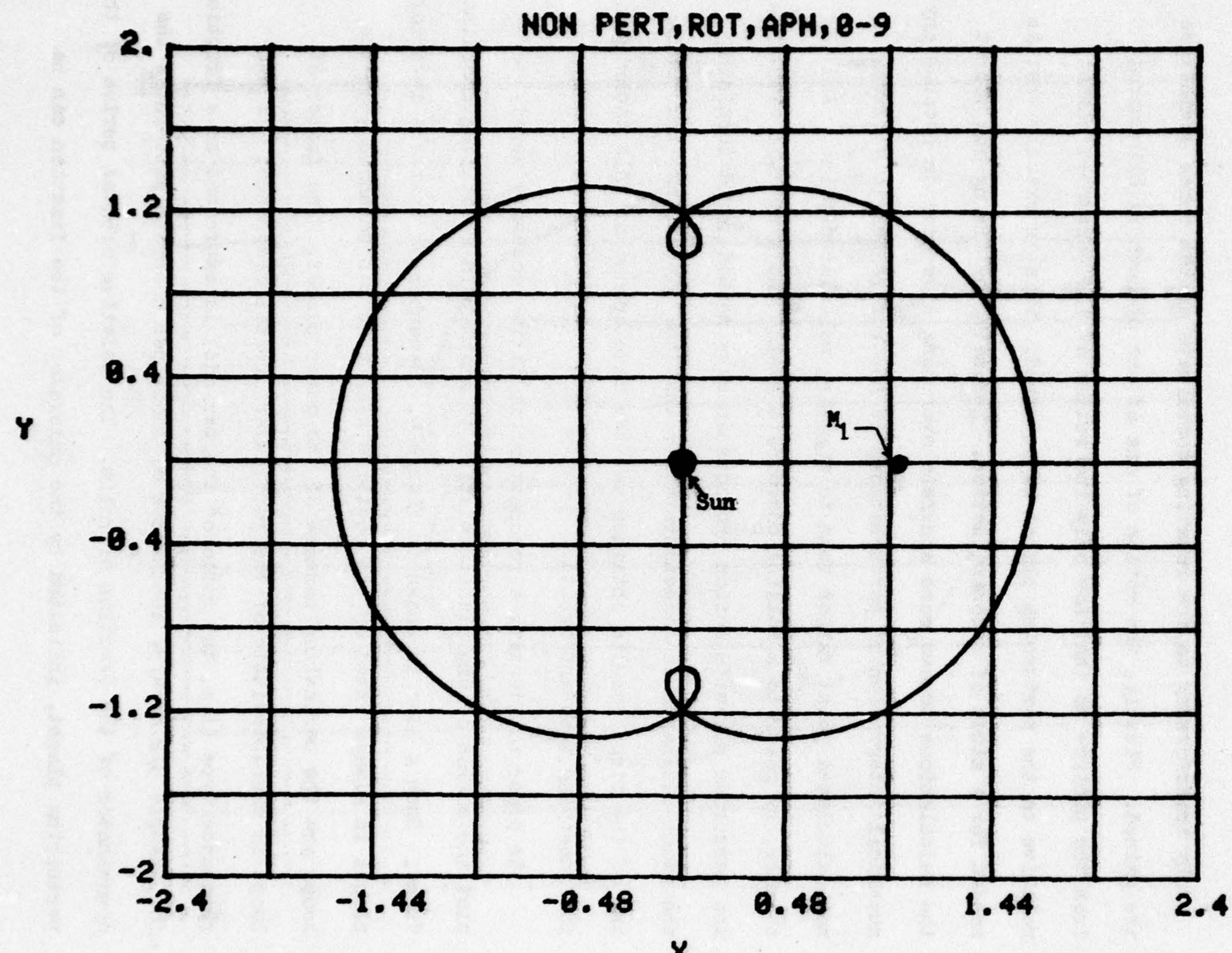


FIGURE 4. Resonant Pattern in a Rotating Coordinate Frame showing a 3/2 Resonance

determined by observing how many orbits it takes for the pattern to complete itself. Thus, if it takes 3 orbits to complete a two loop pattern, a 3/2 resonance is present. This is the case in Figure 4. An easier, although more time consuming method of determining a pattern is to plot the x coordinate of the motion versus time. The time necessary to complete one cycle is the same as the time needed to complete pattern.

By way of illustration the following are a number of different resonant patterns with their x vs. t plots.

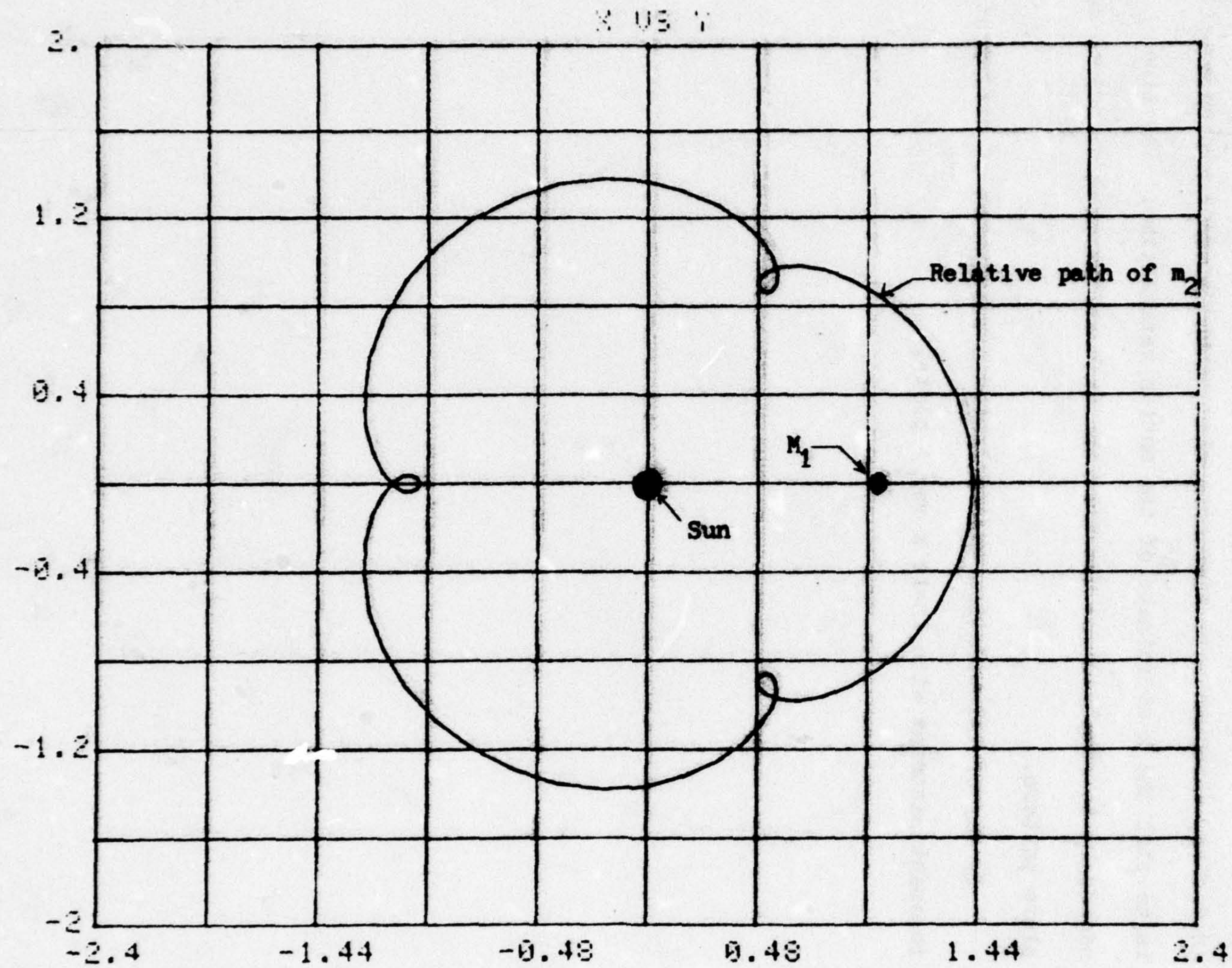


FIGURE 5A. 4/3 Resonance in Rotating Coordinate Frame

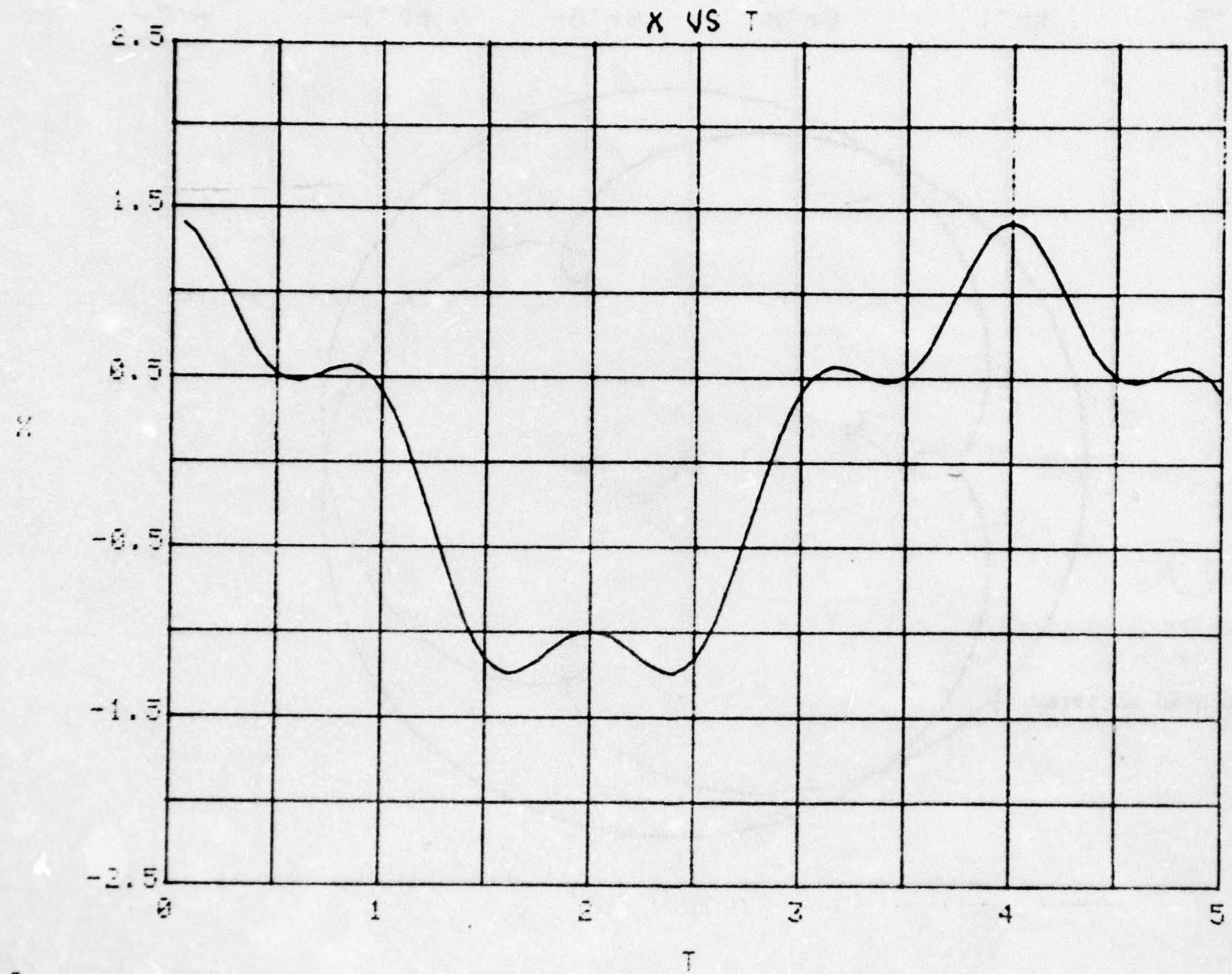


FIGURE 5B. X vs T plot for 4/3 resonance. Note cycle complete at T = 4

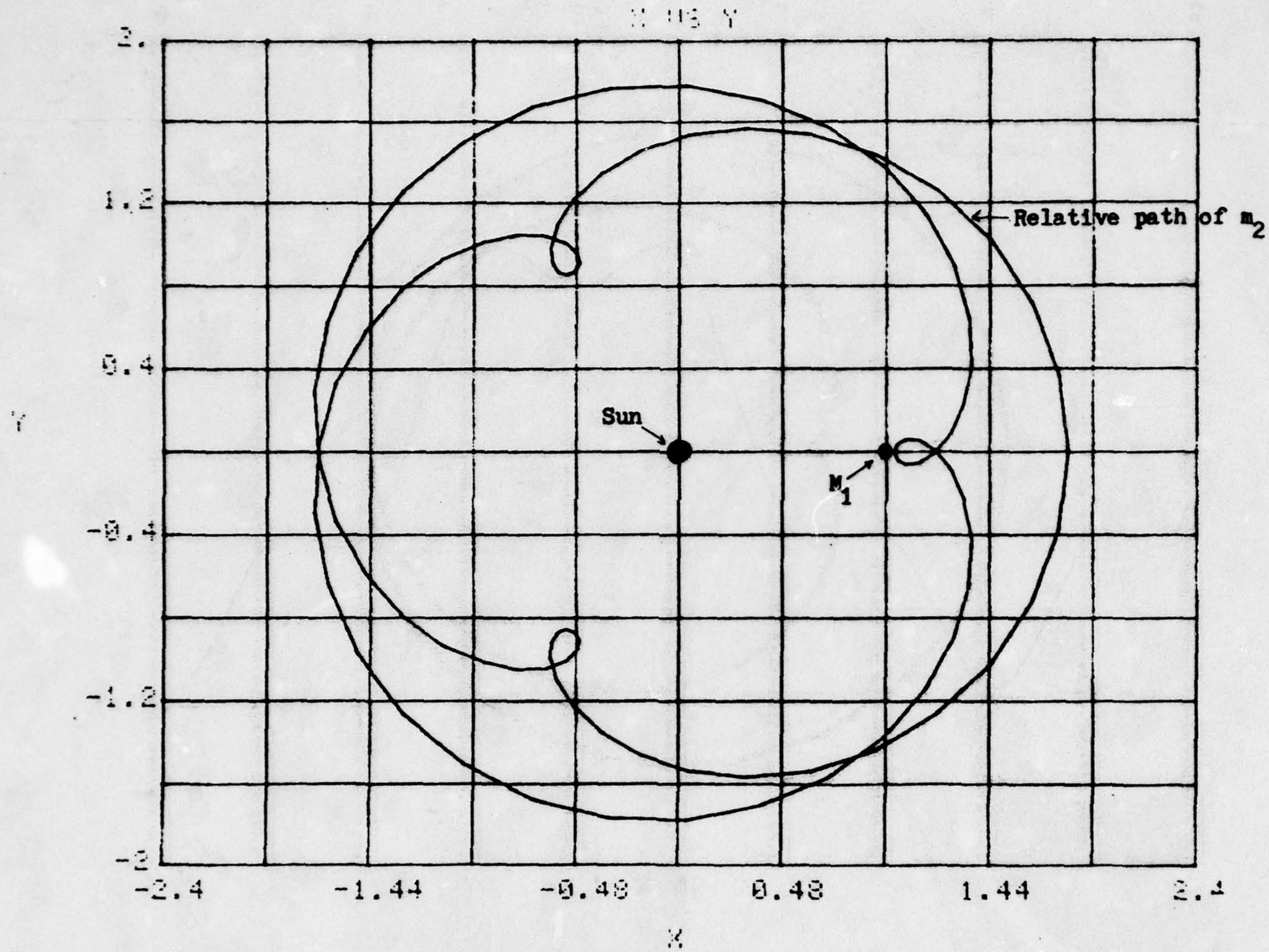


FIGURE 6A. 5/3 Resonance in Rotating Coordinate System

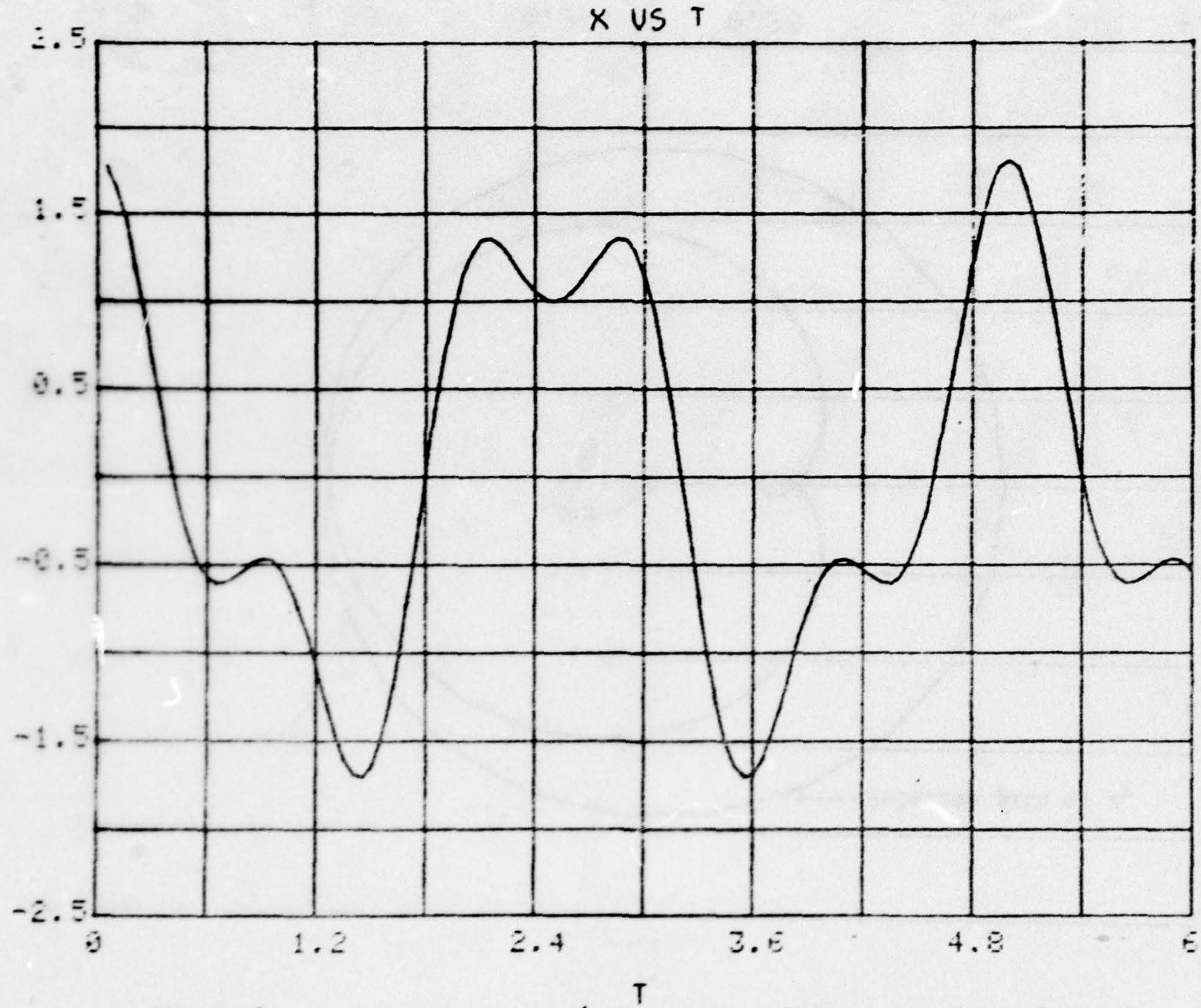


FIGURE 6B X vs T plot for a 5/3 Resonance; Pattern complete at T = 5.

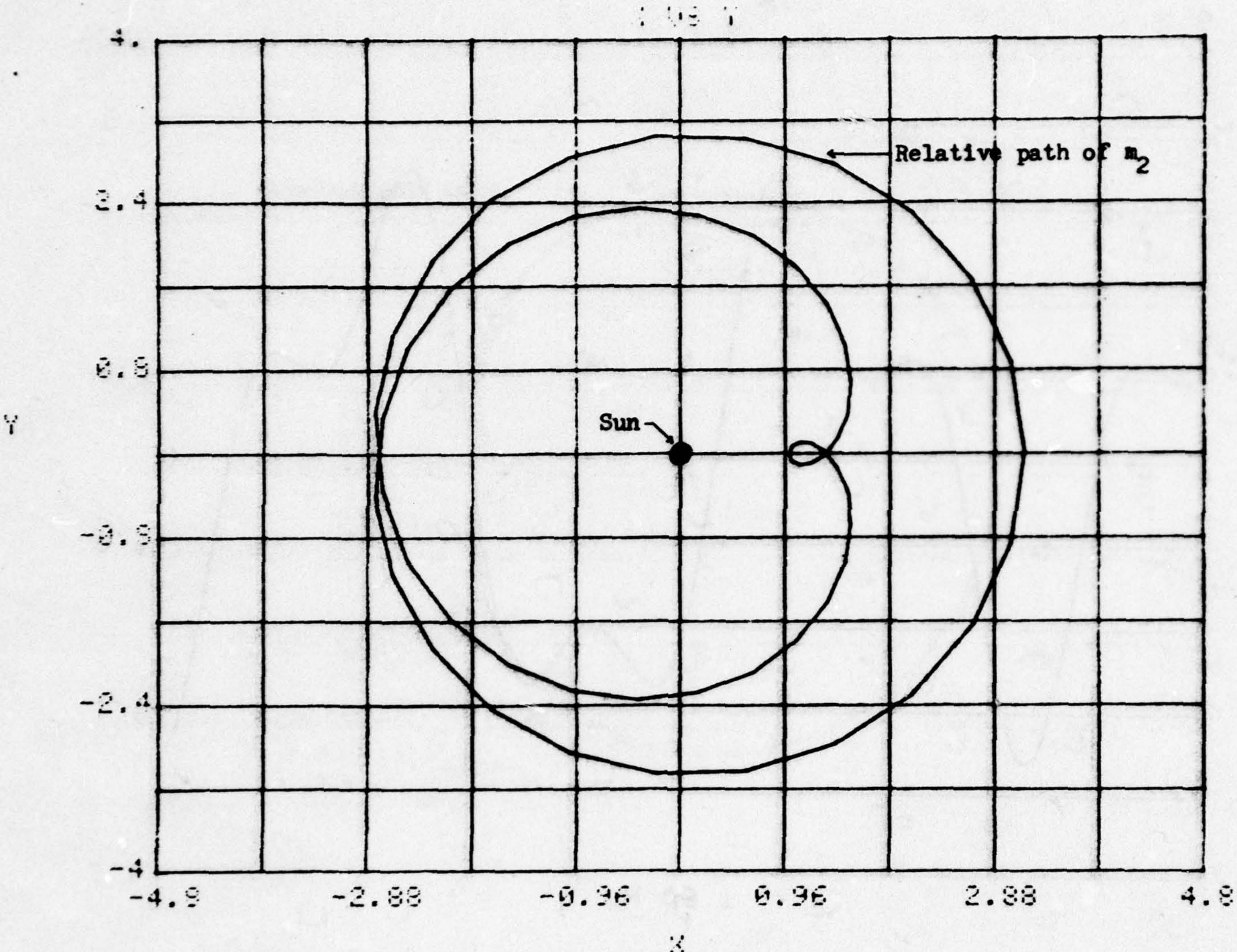


FIGURE 7A. 3/1 Resonance in Rotating Coordinate Frame

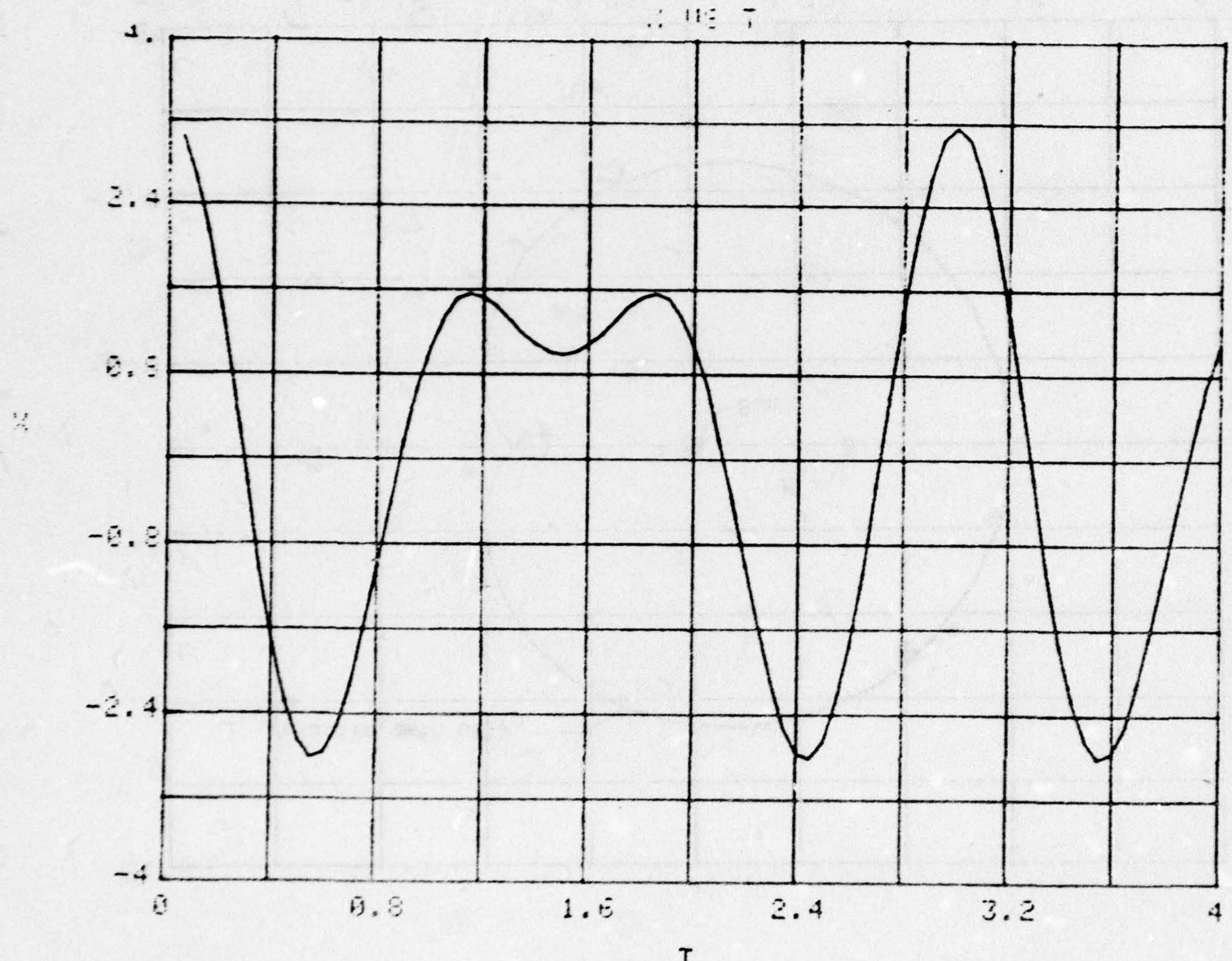


FIGURE 7B. X vs. T for a 3/1 resonance

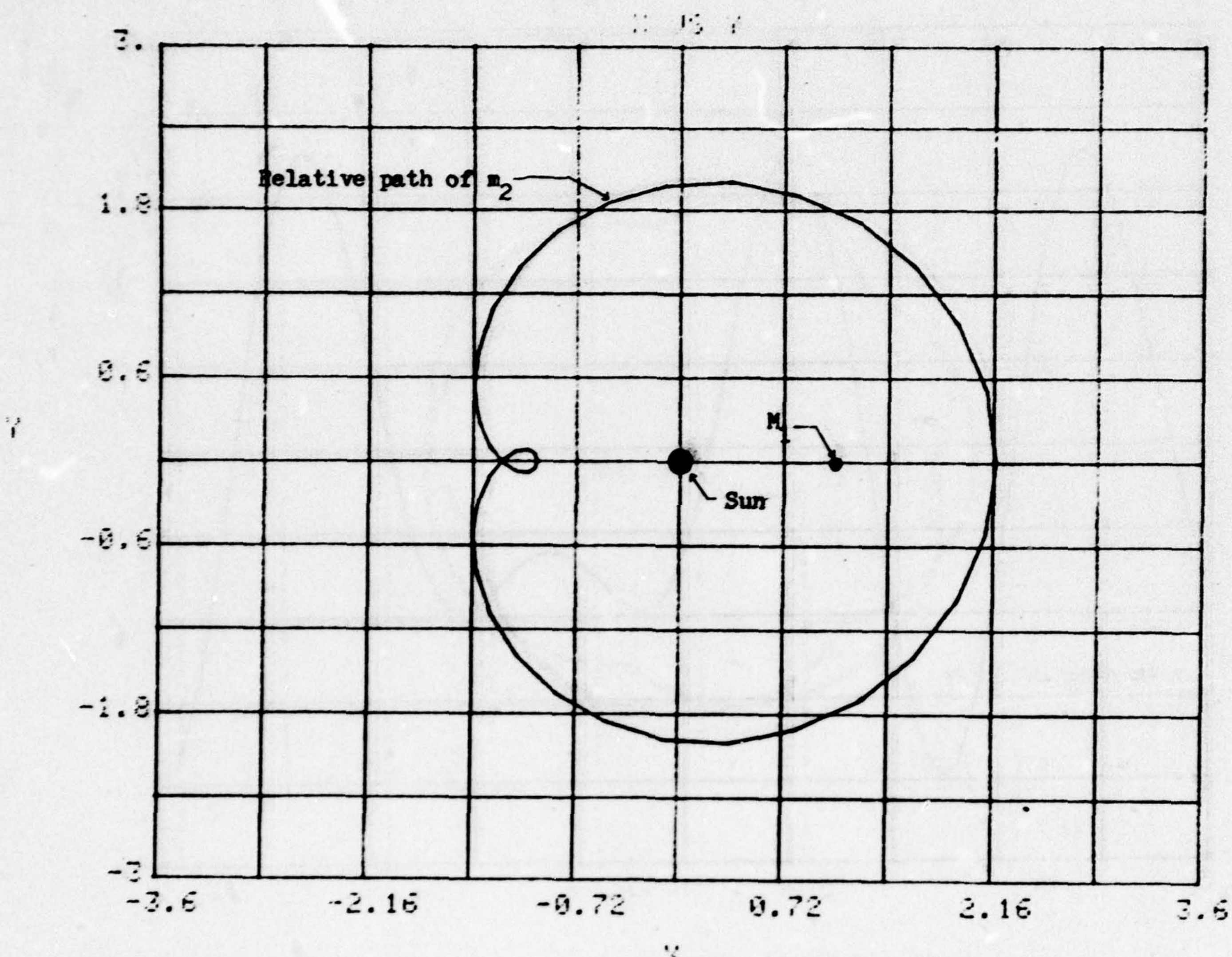


FIGURE 8A. 2/1 Resonance in Rotating Coordinate Frame

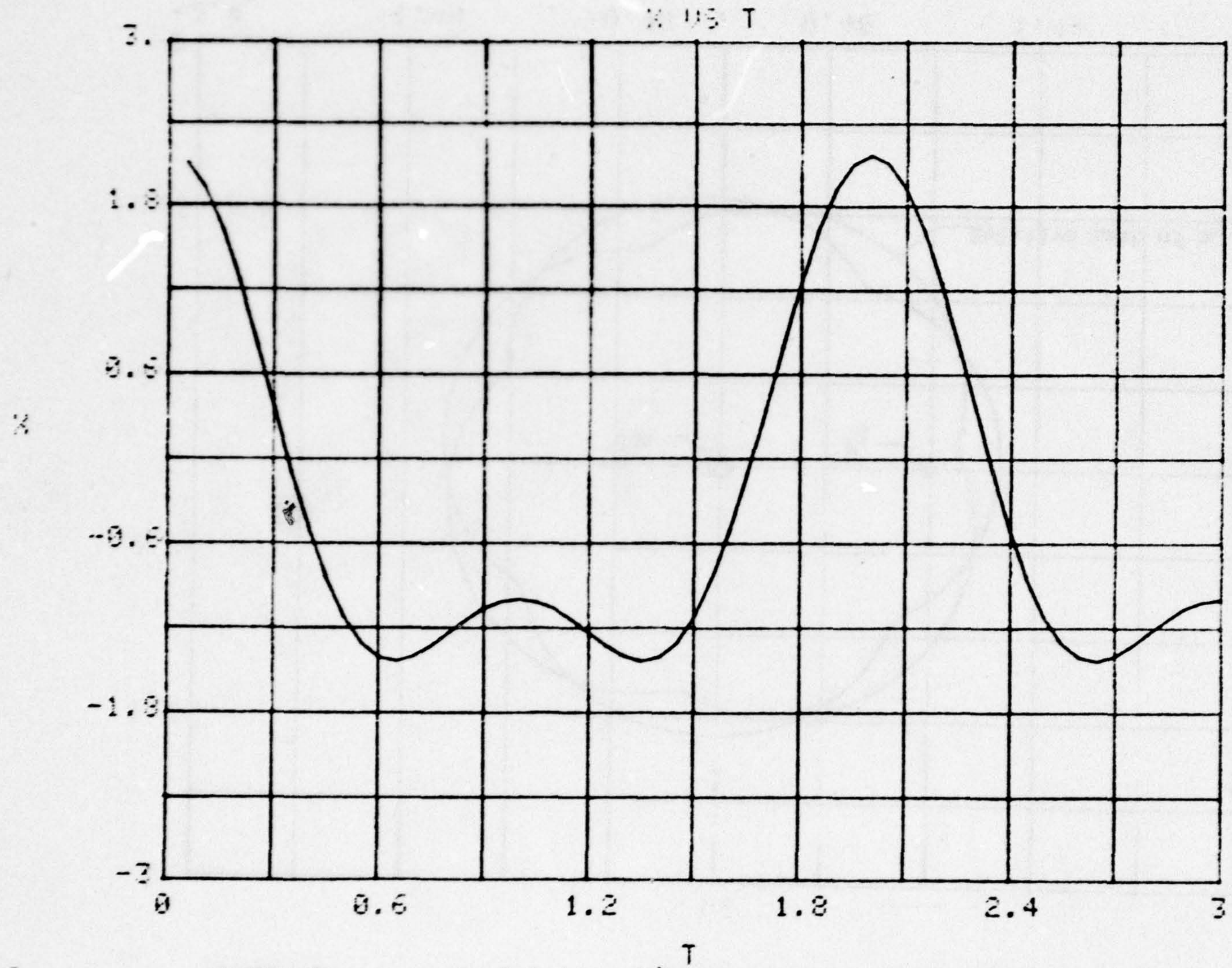


FIGURE 8B. X vs. T plot for a 2/1 Resonance with cycle complete at T = 2

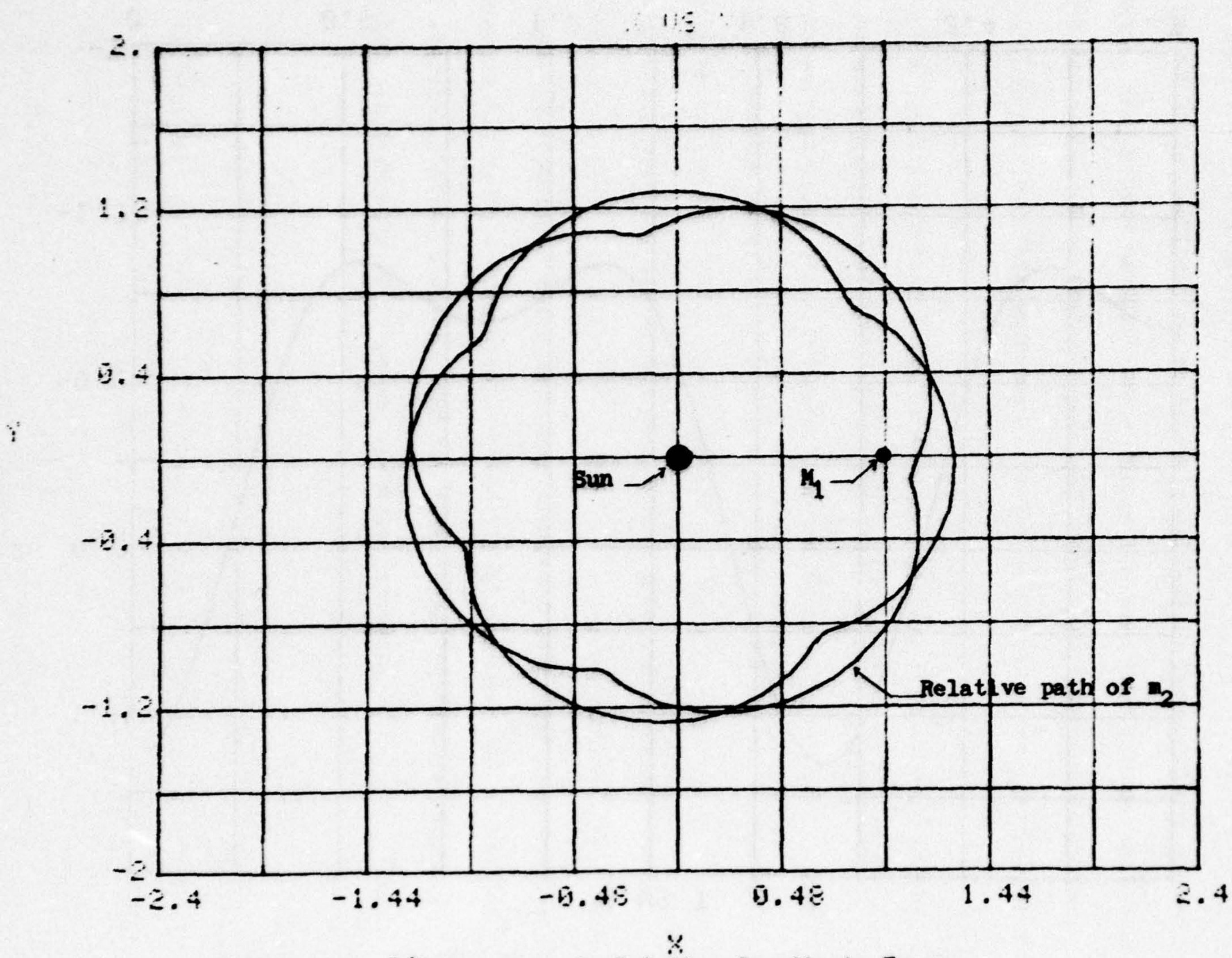


FIGURE 9A. 8/7 Resonance in Rotating Coordinate Frame

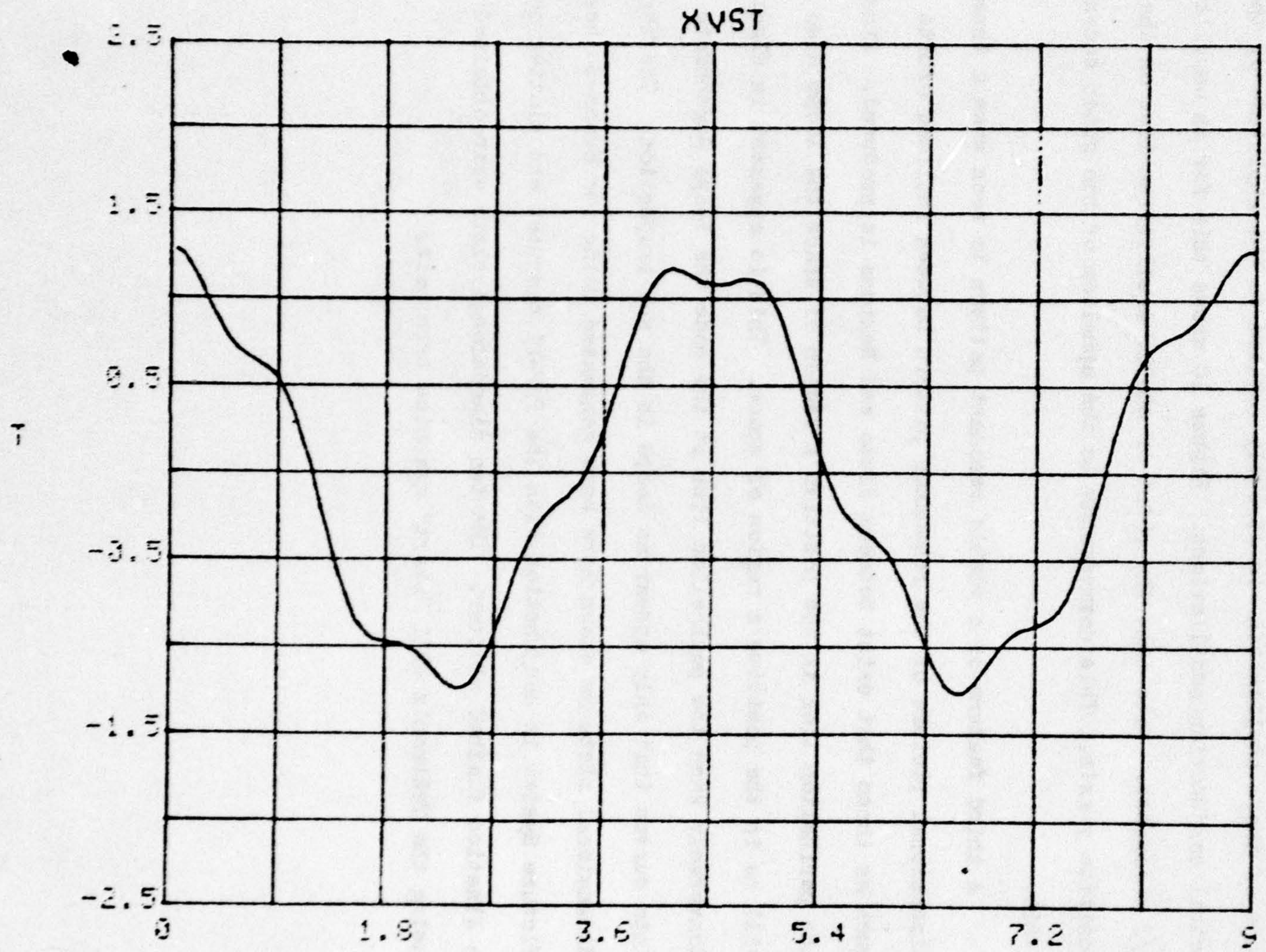


FIGURE 9B. X vs. T plot for an 8/7 Resonance

All the patterns shown are in their stable non-oscillating configurations. In an actual oscillating resonant display the only difference is that the looped pattern itself oscillates about a point. The amplitude of these oscillations is directly related to the amplitude of the actual conjunction oscillations. Figure 10 shows this for an oscillating 3/2 resonance. Note that the point of stable equilibrium lies on the positive y-axis. This corresponds to the aphelion of the outer eccentric orbit.

A third feature of a stable resonant pattern is seen when a three dimensional picture of the resonance pattern between inclined orbits such as those that exist between Pluto and Neptune is produced. Since the perihelion lies in the positive z region of space the loops also will be in the positive z region of space. This is apparent in Figure 11. Conversely when the perihelion lies at the node the loops degenerate into curves that only appear as loops in the x-y projection. The three dimensional pictures shown here were generated using the Evans-Sutherland Picture System in conjunction with the PDP-11 computer and plotted on a XYnetics flatbed plotter. The two dimensional plots were obtained using the Tektronix 4051 "smart" graphics terminals.

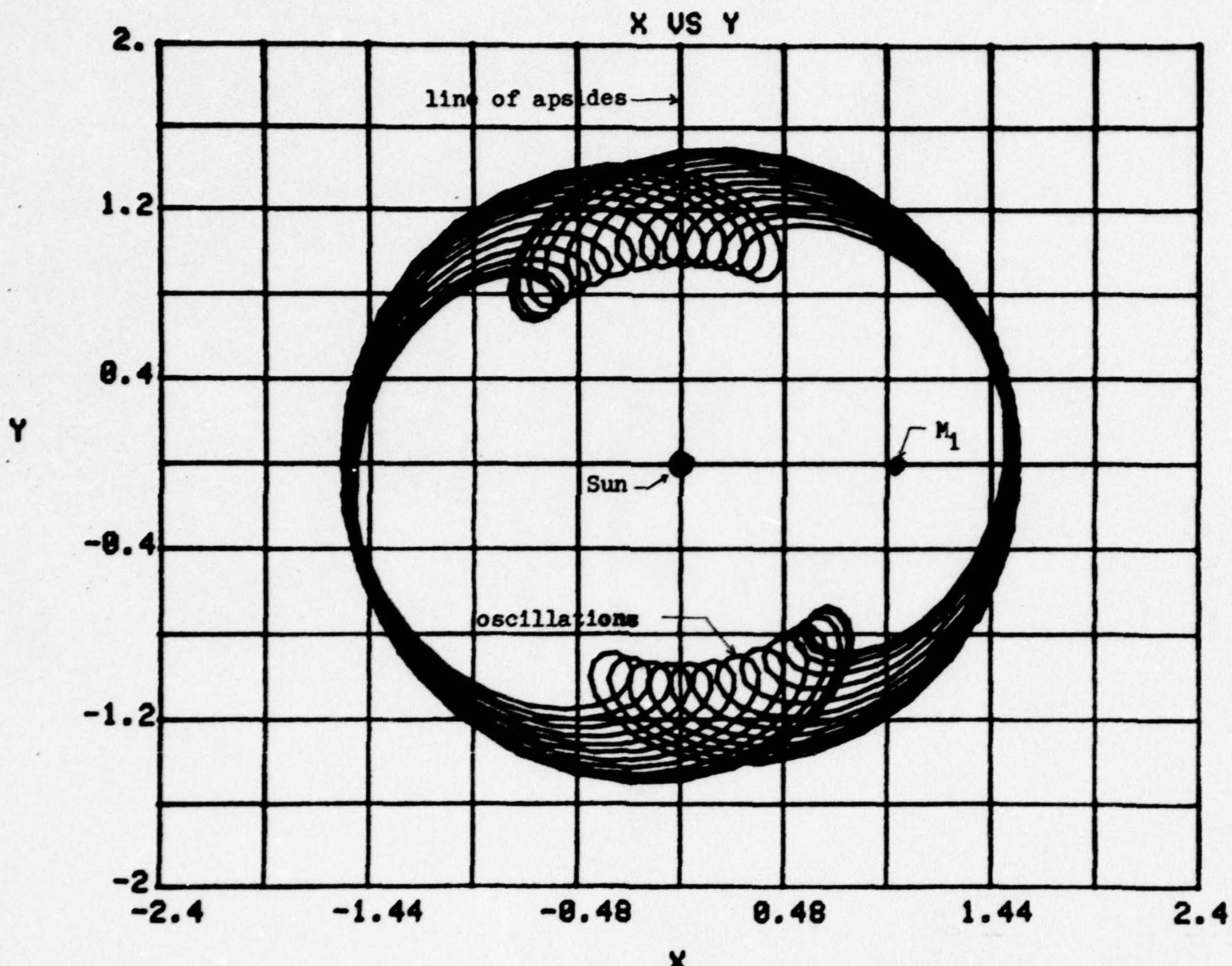


FIGURE 10. Oscillating 3/2 Resonance in Rotating Coordinate System

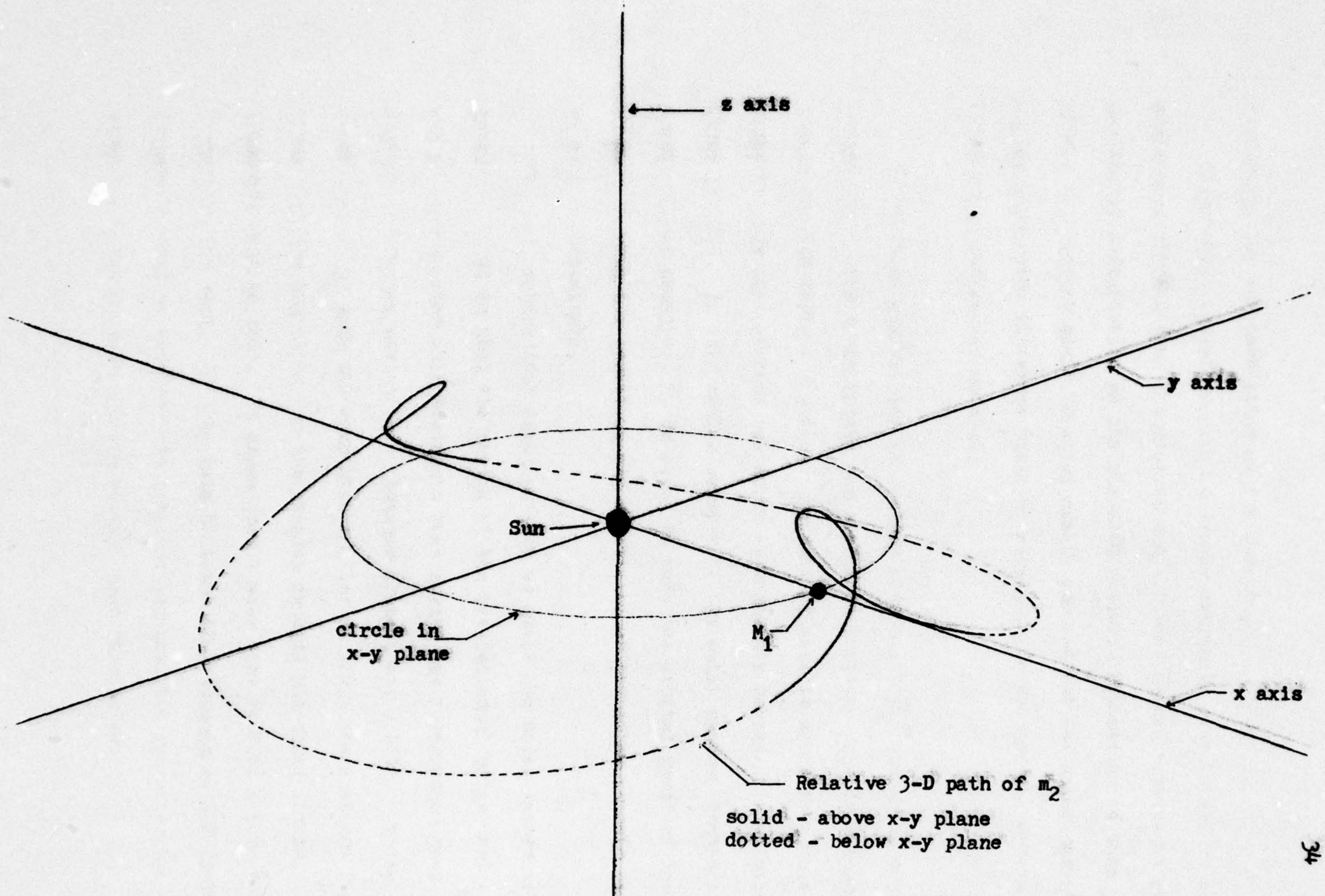


FIGURE 11.

Three Dimensional 3/2 Resonance Pattern formed when orbits are inclined to each other.

Conditions for Initial Formation of a 3/2 Resonance

The present configuration of the Pluto-Neptune resonance is a 3/2 commensurability oscillating about a stable equilibrium point with a period of 20,000 years or about 115 orbits of Neptune. In addition, there are a number of slower oscillations, the most noteworthy of which is a 4 million year or 24,000 orbit oscillation of the argument of the perihelion.[4] The goal then is a formation process starting with Pluto escaping from Neptune and eventually reaching something close to the present configuration cited above. This is done through the use of several computer programs to numerically integrate the positions of the planets of long time periods.

The computer programs used to perform the integrations were all written in BASIC and performed on the U.S. Naval Academy's Dartmouth Time-Sharing System (DTSS). Listings of all programs used are given in Appendices II-IV. The programs were designed for versatility and ease of adaption to new factors. The programs consist of four main parts. First, an input section where variables initialized. Second, the gravitational force of the Sun and any perturbing planets are laid out in the x, y, and z directions. The third and main body of the programs consists of a fourth order Runge-Kutta numerical integration scheme where the forces derived in the second section are evaluated at each of the four steps and from this new positions and velocities of Pluto are computed. The fourth and last section is made up of various output

statements for the tabulation of the results.

Using this type of program it became extremely easy to add or subtract additional planets from the general model. Each planet appears as an additional perturbation to the planet's motion. Proceeding in this manner the planets could be added one by one and the influence each had on the developing resonance could be studied. In the end the entire outer solar system was modeled.

To overcome any problem in dealing with the large distances concerned the entire solar system was scaled so that one unit of distance became the Sun-Neptune distance; one unit of mass is equal to a solar mass; and one unit of time was taken as Neptune's period of revolution about the Sun. In this scaled version of the solar system, Newton's gravitational constant, G, is equal to $4\pi^2$; the orbital speed of Neptune is 2π; and thus the positions and velocities of the other planets must be scaled accordingly.

The one problem that develops when we scale the solar system to Neptune's level is that the period of the revolution of Jupiter and Saturn are so much shorter than Neptune's that during one integration step Jupiter and Saturn move a fair distance along their orbits. On the other hand if we made the step size smaller to limit the distance Jupiter moves then the run times for the program exceeds practical limits. A trade-off must be made. It seems that a step size of 0.01 orbits of Neptune is the best. At this rate it takes 12.8 seconds of CPU (central processor time) to complete one orbit of neptune, 168 years, in the model solar system with all the planets past Mars present.

Assuming Pluto was once in orbit around Neptune, it is advantageous to know some of the particulars of that orbit. Through photometric studies it is possible to determine the rotational period of pluto with great accuracy.[5] If we assume that none of Pluto's rotational energy was lost in any escape then this period should give an approxiamate value for Pluto's period of revolution. Because the mass differential between Neptune and Pluto is so great it is reasonable to assume that it rotated once in every revolution about Neptune much the smae as our Moon does around Earth, always keeping the same side towards the Earth. Knowing this period, P, Kepler's Third Law can be used to compute an orbital radius from Neptune, a,

$$a^3 = \frac{G(M + m)}{4\pi^2} P^2 .$$

This value is $a = 3.8 \times 10^8$ m. The present a for Neptune's other massive moon, Triton, is 3.5×10^8 m. It is then conceivable that a near collision between the two resulted in reversing Triton's motion and ejecting Pluto from orbit. See Figure 12. The research carried out here does not concern itself with the actual collision that produced the escape but only the result of such an escape and whether it was possible for such an escape to evolve into a 3/2 resonance with Neptune.

A wide range of initial escapes were tried. The results were obtained first from a Neptune centered system in which rotation around the Sun was added. (See program NEPTUNE in Appendix II) This introduces a small coriolis term in additon to the Sun's gravity into the force equations. At some point far enough away to avoid scaling problems the coordinates and velocities of Pluto were transformed to the Sun centered reference frame and allowed to continue in its orbit around the Sun.

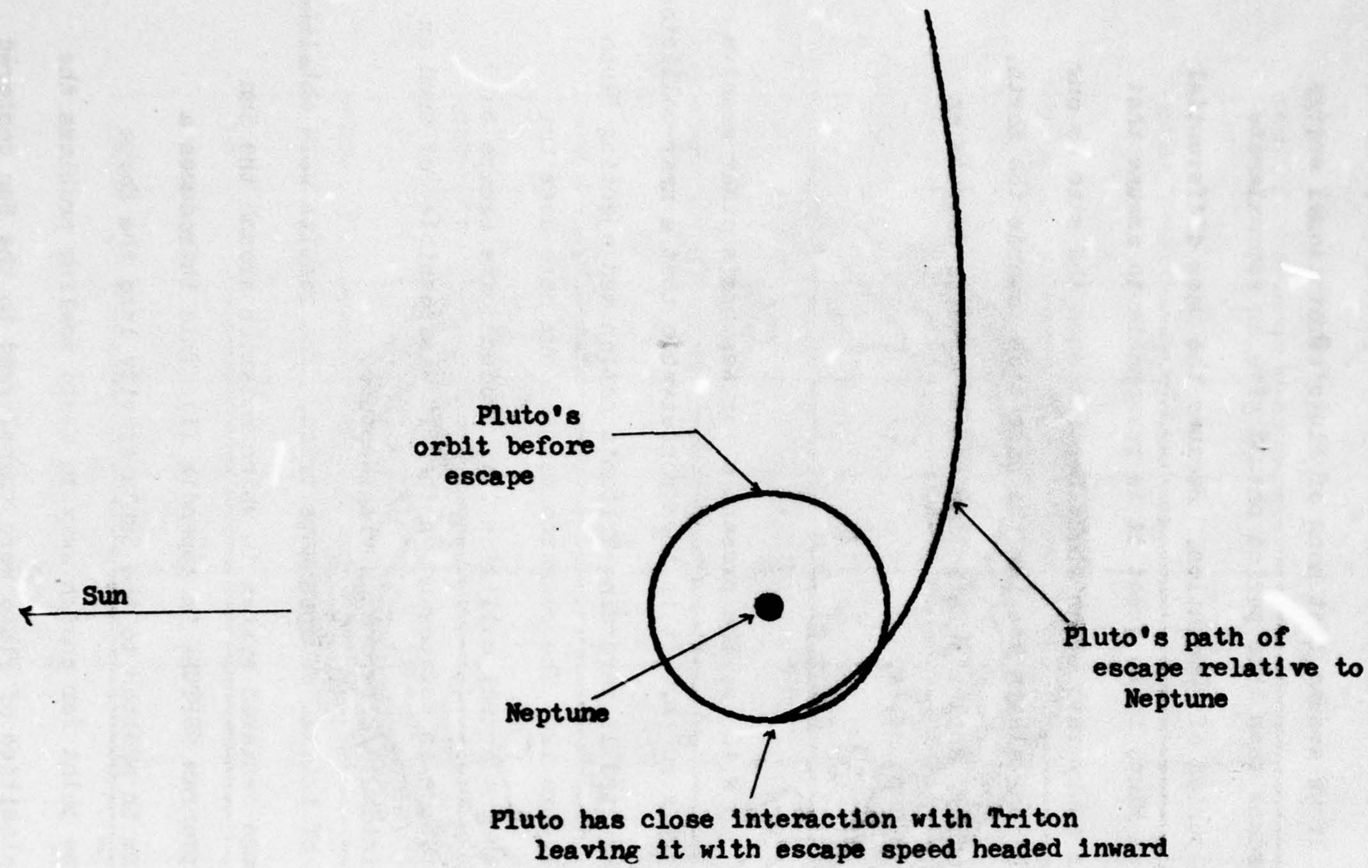


FIGURE 12. Diagram showing path of Pluto's escape from orbit around Neptune through an interaction with Triton.

(See program CONV2 in Appendix IV for conversion factors). By scaling problems we mean that if Pluto's position is transformed into the Sun system too soon its position will be lost in the significant figures that the computer can handle. Therefore, care must be taken to allow Pluto to travel far enough away from Neptune so that there is a significant separation between the two in our scaled solar system.

The results indicate that Pluto will indeed escape if given even a sizeable fraction of the necessary escape speed. A resonance between Pluto and Neptune will be formed in a majority of the cases. These range from an unstable 8/7 to a very stable 4/3 resonance. See Appendix I for a discussion of relative orbital stability.

The 3/2 resonance will be established if a close interaction between Pluto and Triton leaves Pluto with near escape speed heading in a direction that is 13^{o} to 19^{o} inwards towards Neptune as measured from the tangent to the original circular orbit. Pluto, given these conditions immediately starts to exhibit the characteristics of resonant oscillation and stability mentioned before.

First, since the escape must take place in Neptune's orbit the escape point is both the node, the point of conjunction, and near the perihelion of Pluto's new orbit. It was shown that if the conjunction occurs anywhere other than the the aphelion the resonance will begin to oscillate about the aphelion. For the case shown in Figure 13 it can be seen that the resonance is initially a 3/2 resonance. The oscillations are very apparent. Because they start near the perihelion the amplitude is large. Figures 13, 14, and 15 show the period from escape at t = 0 out to 135 orbits of Neptune. It can been seen that the oscillations are slightly

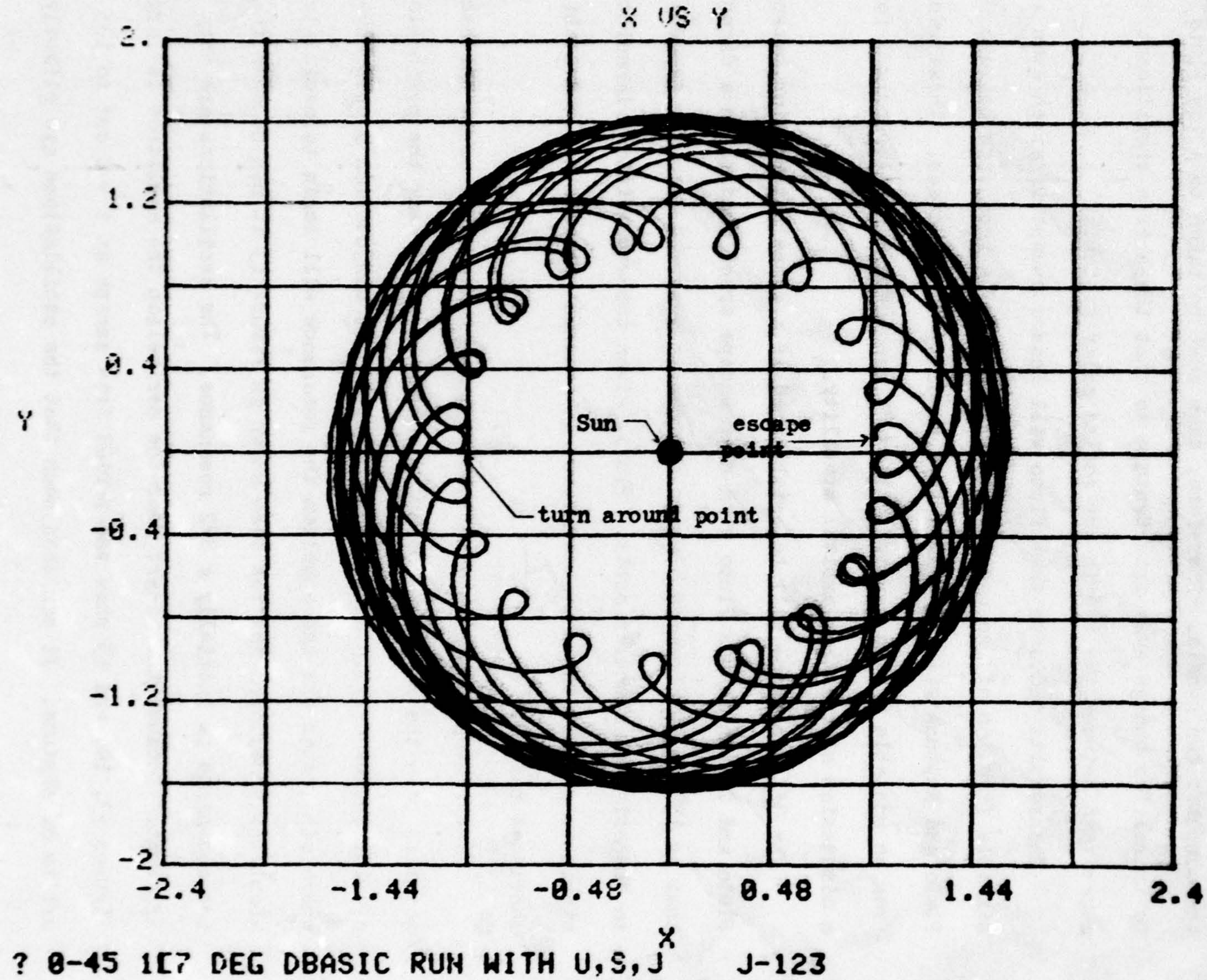


FIGURE 13. Oscillating 3/2 Resonance from 0 to 45 orbits after Pluto's escape from Neptune

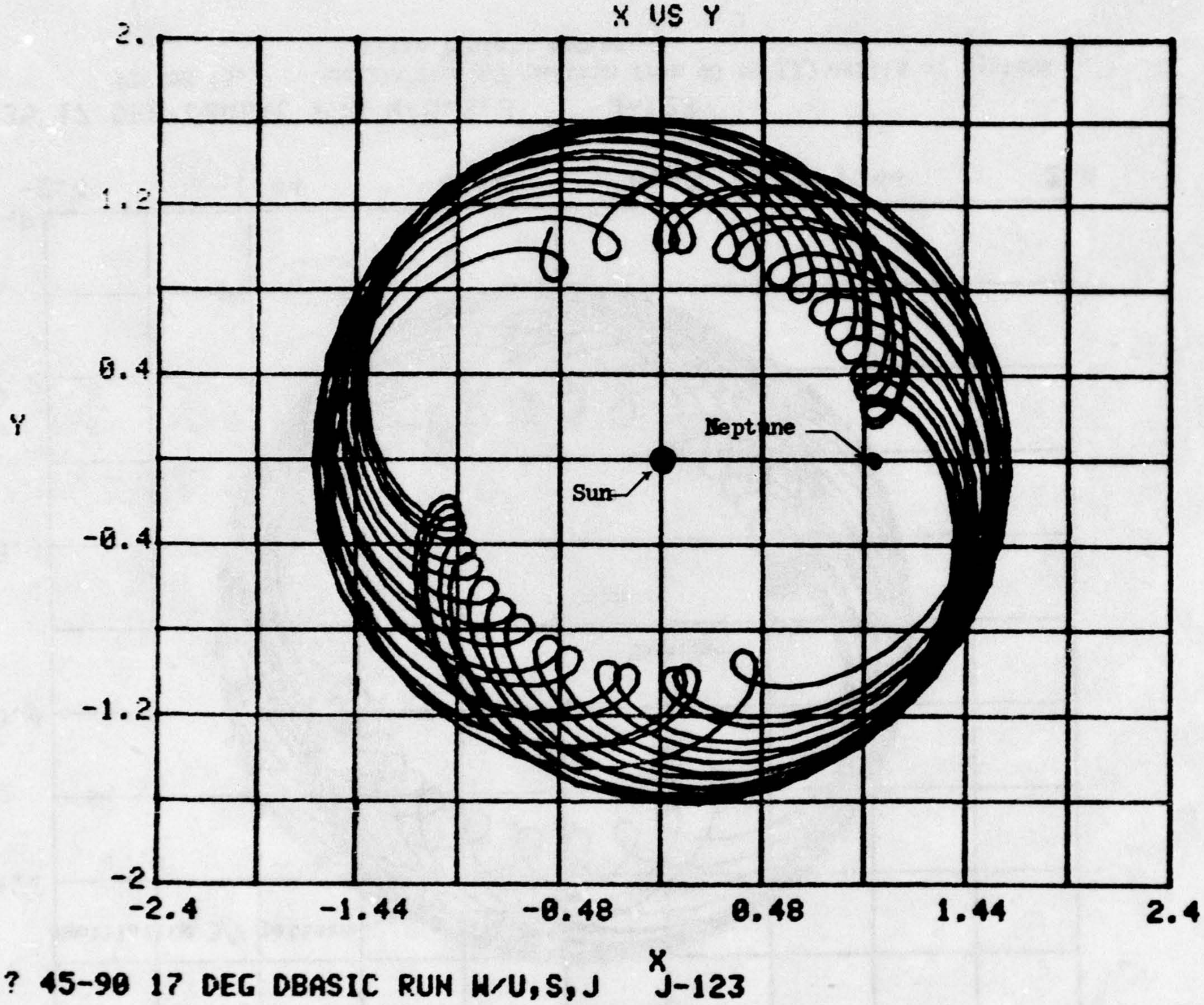


FIGURE 14. Pluto's 3/2 resonance pattern from 45 to 90 orbits of Neptune (15,000 yrs.) after the escape.

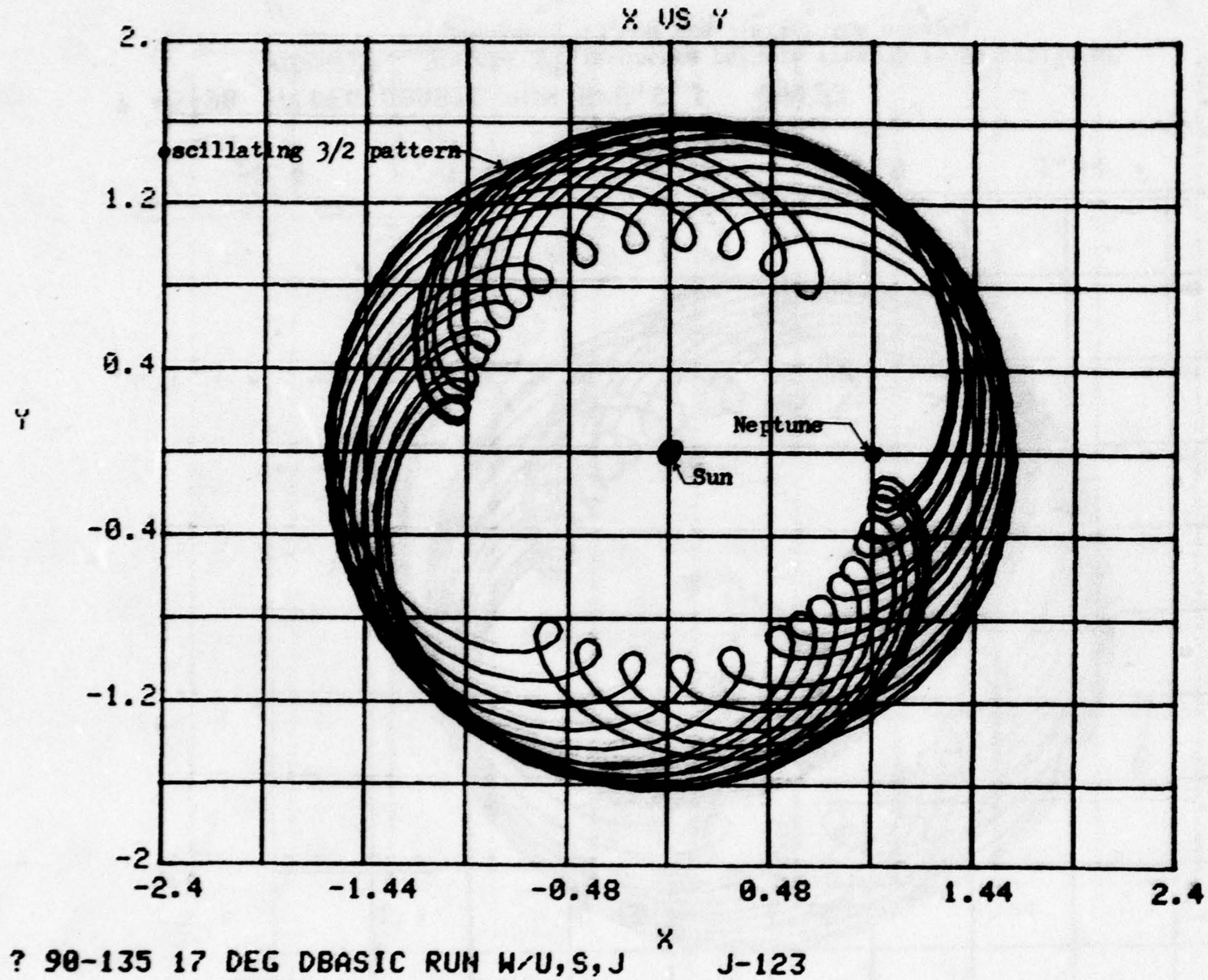
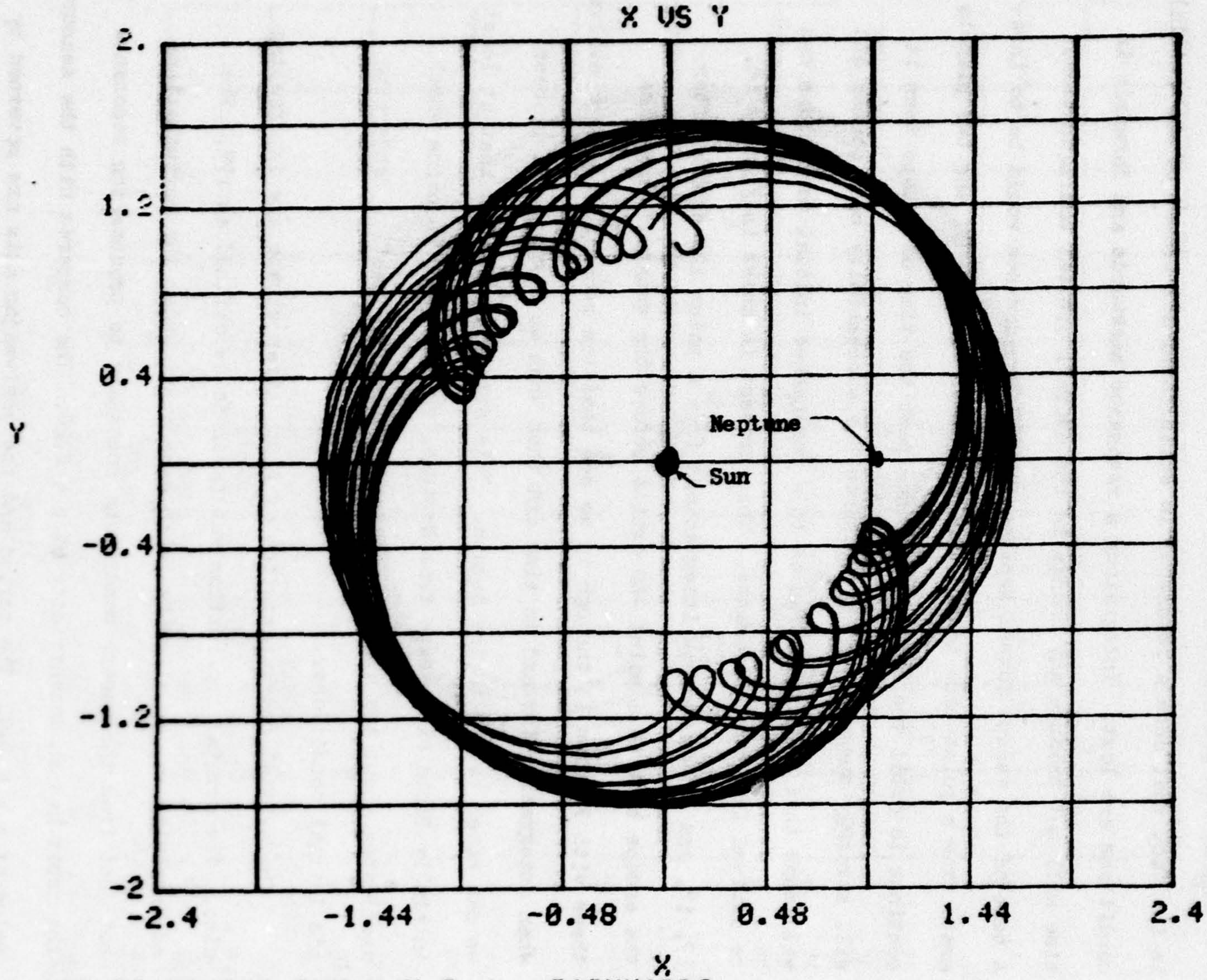


FIGURE 15. Oscillating 3/2 pattern from 90 to 135 orbits of Neptune after Pluto's escape.

damped and pluto at perihelion does not quite return to the node. Thus, at least for a short period the resonance is stable.

One of the primary characteristics of an orbital resonance discussed is the fact that once a resonance is established all traces of any initial conditions are lost. Integrating a resonance backwards and forwards in time will not produce any change in the overall orbital configuration. A test of the escape formed Neptune-Pluto 3/2 resonance would be to integrate from a point back towards the moment of escape. If the two planets continue to orbit the Sun in resonance past the time of escape then it will strongly suggest that a resonance has successfully been formed and will show that no information as to a resonances initial conditions can be derived from that resonance. This process is shown in Figures 16, 17, 18, and 19 which show integrations from a point 120 orbits after the escape back to a point -60 orbits before the escape. Comparing these with Figures 13 through 15 we see that the resonance pattern evolves when integrating forward in time and then when we reverse the process we can never return to the escape. This strongly suggests that at least initially Pluto can escape from Neptune, enter a 3/2 resonance which oscillates about the aphelion and contains no information concerning its initial conditions.

The orbital characteristics of Pluto after escape are interesting also. The orbital inclination was taken to be defined as 17°. The eccentricity varies slightly with each orbit under the perturbations but just from the escape conditions necessary to produce the resonance the orbit has an eccentricity of $e = 0.24$. The compares with the measured value of $e = 0.249$. The period and the semi-major axis are governed by



? 120-75 DBASIC RUN W/U,S,J BACKWARDS

FIGURE 16. Pluto's 3/2 resonance being run <u>backwards</u> in time towards the point of escape

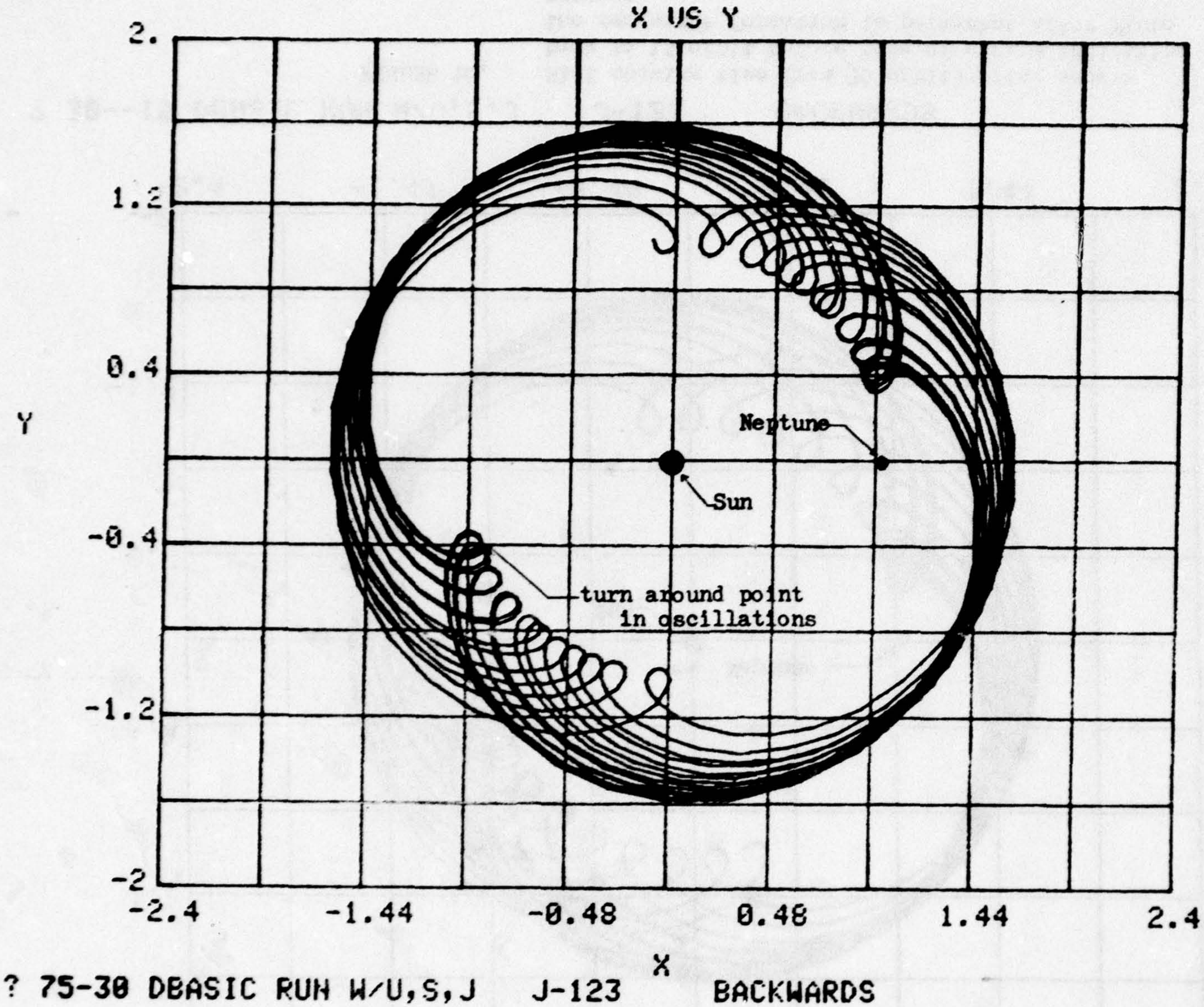


FIGURE 17. Oscillating 3/2 Pluto-Neptune resonance running <u>backwards</u> towards escape.

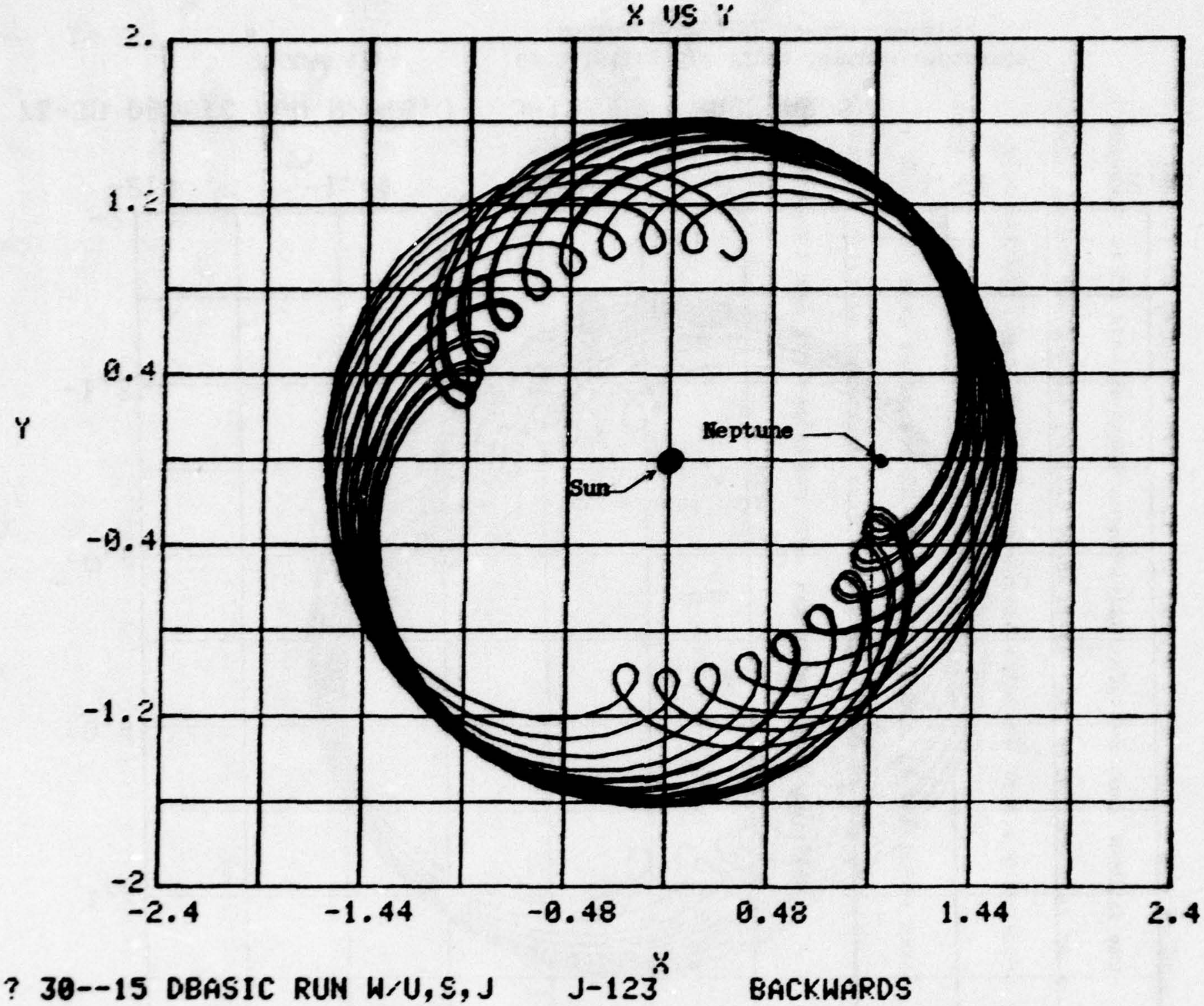


FIGURE 18. Plot showing time from 30 orbits after escape back to 15 orbit before time of escape indicating the resonance formation is permanent after Pluto escapes.

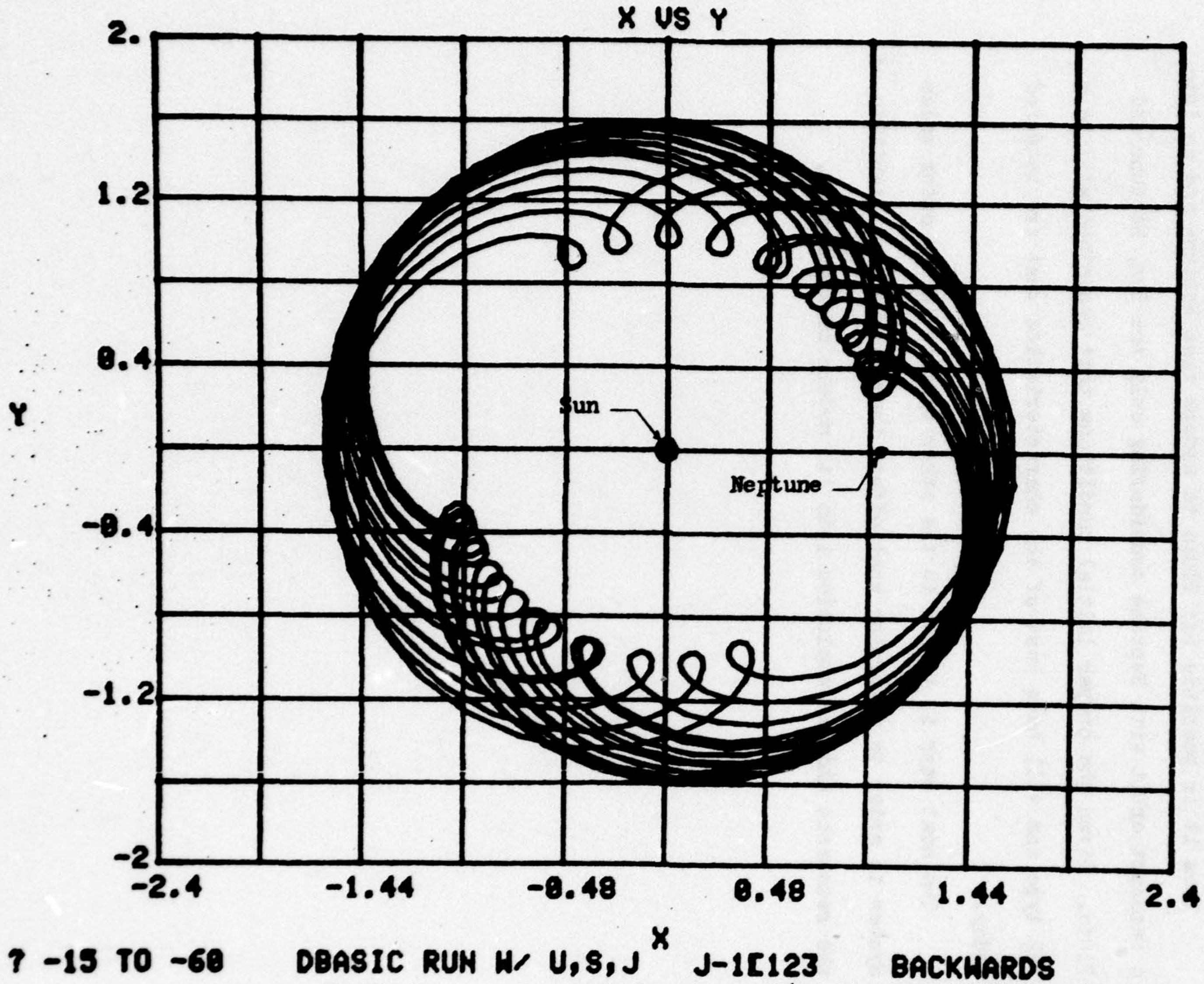


FIGURE 19. Plot showing 3/2 resonance still oscillating backwards in time before the escape indicating that the initial conditions are lost in a resonance.

Kepler's Third Law and with the conditions necessary for resonance we arrive at values for period and semi-major axis that are close to the present measured values. See Figure 20.

Thus it is possible for Pluto to escape from Neptune and go into a resonant orbit with Neptune considering only the Sun, Neptune and Pluto. Given the proper initial conditions that resonance will be a 3/2 type and will have most of the characteristics that are measured today.

The next step is to add in the other planets in the outer solar system in order to determine what effect they have on the stability of the resonance and its evolution into its modern configuration.

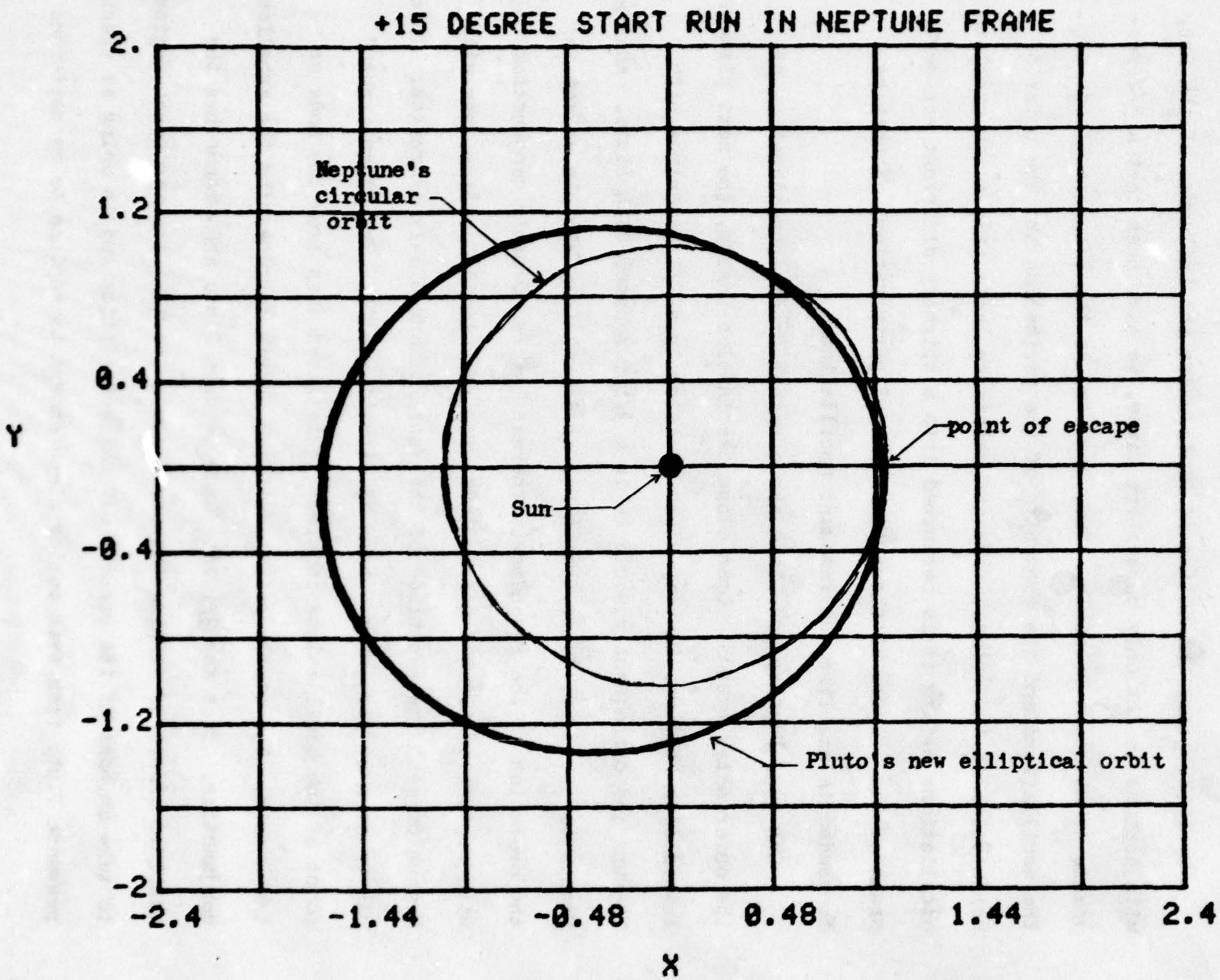


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FIGURE 20. Plot showing Pluto's orbit about the Sun after escape from Neptune.

Effects of Uranus on the 3/2 Resonance

By integrating the motion of Pluto under the influence of the Sun, with Neptune as the only perturbing force, we have seen that a 3/2 resonance is at least possible. Due, however, to the large amplitude of the oscillations and the proximity of the perihelion and the point of conjunction to the orbital node the resonance lasts only a few complete oscillations before it is perturbed into a slightly different non-resonant orbit. Figure 21 shows this sort of perturbation. Something is needed to stablize the resonant oscillations.

The Sun, Neptune and Pluto are a rather crude approxiamation of the outer solar system. Upon close examination Uranus, the next planet inward from Neptune, is very close to being in a 2/1 resonance with Neptune and consequently would be in a 3/1 resonance with Pluto. Adding Uranus to the model solar system has an immediately visible effect on the evolution of the resonance. Whereas before, Pluto's conjunction with Neptune oscillated nearly 180° with very little damping, now with Uranus present the amplitude of the oscillations slowly decreases. This can be seen by comparing Figure 22 with Figure 15. The "turn-around" point of the oscillations is clearly displaced away from the node on the x-axis. The stable point is on the y-axis representing the aphelion conjunction. Thus the 3/1 resonance between Pluto and Uranus has the combined effect of damping the oscillations so that the resonance begins to take on some of its present day characteristics and in doing so Uranus prevents Pluto from ever coming close enough to Neptune to be perturbed

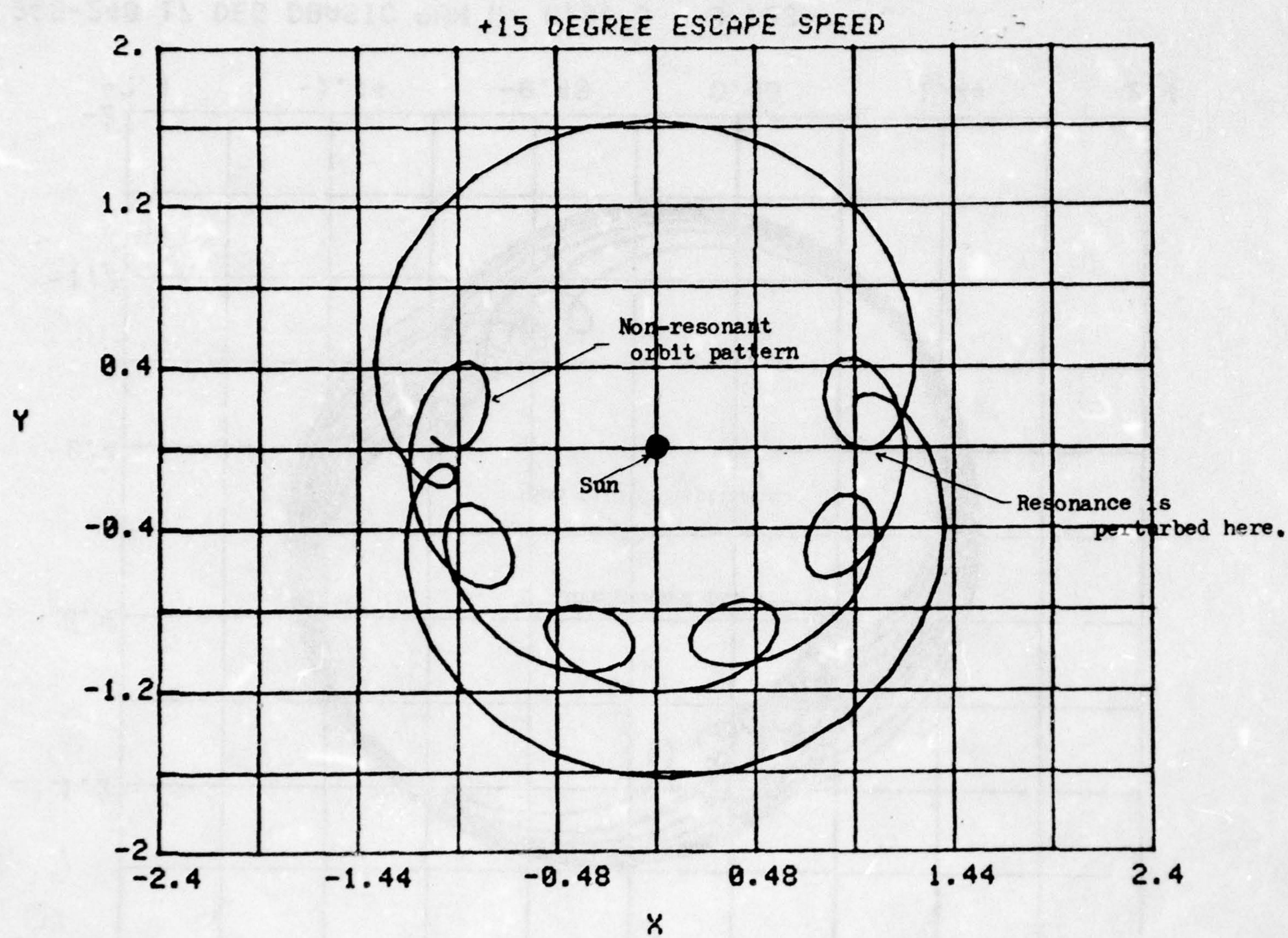

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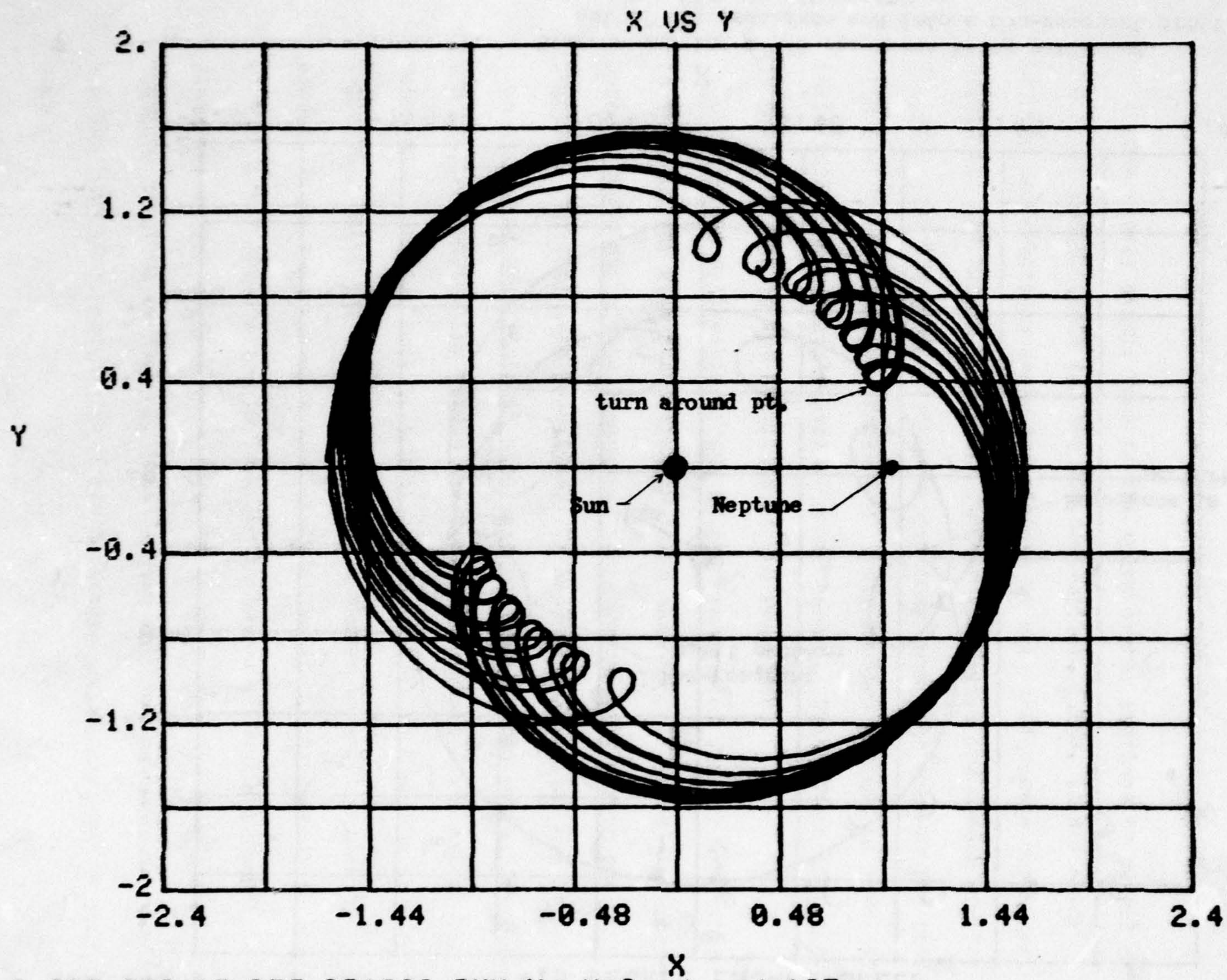
FIGURE 21. Diagram showing a 3/2 resonance being perturbed out of the resonance and into a non-resonant orbit. Rotating Coordinate Frame



? 200-240 17 DEG DBASIC RUN W/ U,S, J J-123

FIGURE 22. 3/2 Resonance 240 orbits after escape: Note location of turn around point relative to Neptune.

out of the orbital resonance. This is the situation today when Pluto never comes closer than 16 A.U. to Neptune.[6] We now have an escape formed 3/2 resonance which is independent of the initial start that is stable with Uranus and Neptune for approxiamately 1340 orbits of Neptune or the equivalent of 225,000 years. After that the resonance begins to show signs of decay and the amplitude of the oscillations increases until Pluto is perturbed out of the resonance. Long term perturbations appear to be needed to give the orbit additional stability.

Long Term Effects of Saturn and Jupiter

Even a very simple look at the solar system suffices to reveal that the major constituents are the Sun and Jupiter. Thus, in any study involving perturbations among the planets we must include the influence of Jupiter and Saturn. Although Jupiter and Saturn are a much greater distance from the Pluto-Neptune system they more than make up for this with their large masses. Because of their shorter periods of revolution their influence is mainly over a longer period. This is exactly what is needed to add to the stability of the resonance.

The first noticeable effect of the presence of Jupiter and Saturn is that Jupiter with its extremely large mass can, if positioned correctly, perturb Pluto out of the resonance. This occurs only during the initial stages of the resonance formation. If Neptune and Jupiter are in conjunction at a time when Pluto is also near conjunction and perihelion passage the combined perturbing forces are enough to perturb Pluto out of the resonance. Jupiter must be positioned so that it will not be in such a position until the resonance has had a chance to shift enough to prevent Pluto from approaching closely to Neptune. If Jupiter is placed so that after Pluto's escape from the influence of Neptune, Jupiter and Pluto are in conjunction when Pluto is at or reasonably near its greatest distance relative to Neptune, then Jupiter will not perturb the resonance and the long term perturbations due to Saturn and Jupiter can be fully evaluated.

With the addition of Jupiter and Saturn we now have a complete computer model of the outer solar system and a very good approxiamation of the entire solar system due to the small relative masses of the inner terrestrial planets. We can now study the evolution of the Pluto-Neptune resonance in detail.

It was found that if the escape of Pluto occurs in such a manner that when Pluto is initially at its farthest relative distance from Neptune it is near conjunction with Jupiter then a stable 3/2 resonance can develop. Pluto's new orbit has an eccentricity roughly equal to 0.24. The resonance oscillates with a very large amplitude about the aphelion conjunction point. This pattern was found to be constant for nearly 7000 orbits of Neptune which is the equivalent of 1,176,000 years. There are no signs that the resonance can be perturbed out of this configuration. The oscillations have been damped enough so that they no longer come near Neptune. In addition, as was mentioned before, reversing the integration produces no changes in the resonance. Thus, with all the planets present we are able to form a true resonance between Pluto and Neptune.

One additional feature of the resonance was identified as a result of long term integrations. Examining the velocity in the z-direction at the moment of perihelion passage enables us to locate the perihelion. At escape the perihelion z-velocity was near its maximum value indicating that the perihelion lay near the node between Pluto's and Neptune's orbits. As the integrations progressed the z-velocity at perihelion decreased to zero and then began to increase in the negative direction. This indicated that the perihelion was precessing away from the node

and up away from the plane of the ecliptic. Current studies have shown that the perihelion oscillates with a period of four million years about a point 90° away from the orbital node.[7] Upon carrying out the integration to slightly ower one million years we found that the perihelion was circulating instead of oscillating. There have been studies that show an circulating perihelion but they do not include Jupiter and Saturn in their analysis of the Pluto-Neptune system.[8] The fact that the perihelion circulates rather than oscillates does not invalidate the other results. It is certainly possible that continued integrations or more refined calculations of the perturbations due to Jupiter and Saturn might change the circulating perihelion to its present oscillating form. One million years is a long time on man's sclae but a mere flicker astronomically.

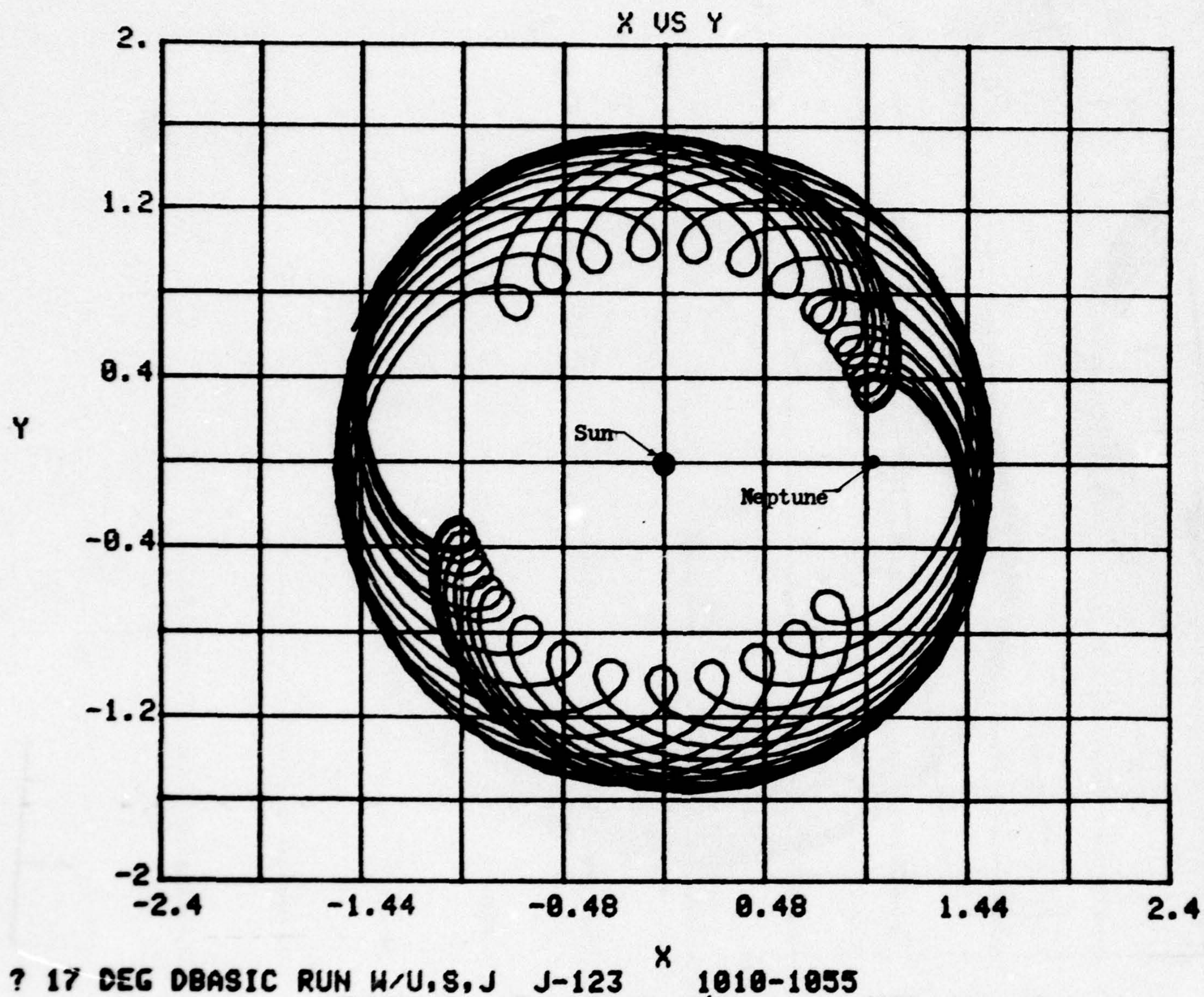
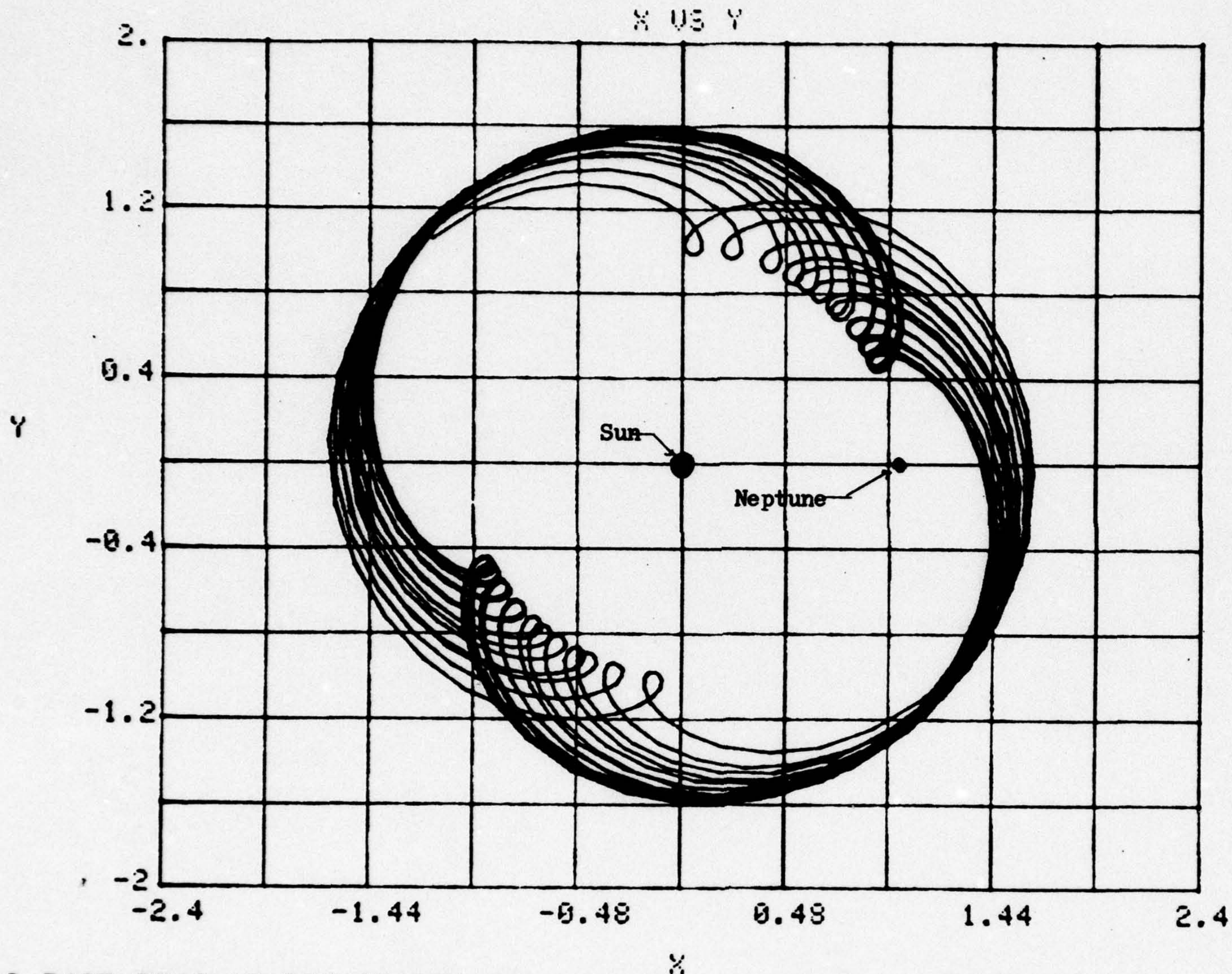


FIGURE 23. Pluto-Neptune 3/2 resonance 1055 orbits after escape



? 5495-5540 17 DEG DBASIC RUN W/U.S,J   J-123   0=3350

FIGURE 25. Pluto and Neptune 3/2 resonance 924,000 years after Pluto escape from Neptune

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Conclusions

It has been shown that it is possible that if Pluto and Triton had a close encounter resulting in Pluto attaining escape velocity directed approxiamately 17^{o} inward from its previous orbit then a stable 3/2 oscillating resonance will evolve. Pluto will orbit the Sun in an orbit that has all the characteristics of its present day orbit. The resonance has been found to be stable for over one million years with no evidence to suggest that it will not continue to remain stable. Pluto's perihelion was found to circulate rather than oscillate but this is explainable as a result of the perturbations from Jupiter and Saturn.

The objection may arise that we have placed too many restrictions on the solution and that the solution may have been forced. This ignores the fact that any resonance in itself is a special case and a 3/2 resonance is a special case of that. Many initial conditions were found that allowed Pluto to escape and orbit the Sun without entering a 3/2 resonance. But Pluto is in a 3/2 resonance today so we had to look for the particular conditions that gave us a 3/2 resonance.

This project was carried out for the purpose of determining whether it was possible for Pluto to have escaped from Neptune and evolve into 3/2 resonance. Such a possiblity was found and in doing so we have eliminated the major objection to the moon hypothesis as to Pluto's origin. That objection was that since Neptune and Pluto are in resonance they would never come close enough to one another for Pluto to have been a moon. We have shown that an escape will produce a resonance and once the

resonance is established all the information regarding the initial start is lost. The evolution of the resonance keeps Pluto and Neptune separated and has all the modern characteristics that have been observed in the system.

Footnotes

1. S. J. Peale, "Orbital Resonances in the Solar System", Annual Review of Astronomy and Astrophysics, Vol. 14, 1976, Annual Reviews, Inc., p. 215.

2. Elske v.P. Smith and Kenneth C. Jacobs, Introductory Astronomy and Astrophysics, W. B. Saunders Company, Philadelphia, PA., 1973, p. 111.

3. J. G. Williams and G. S. Benson, "Resonances in the Neptune-Pluto System", The Astronomical Journal, Volume 76, No. 2, March, 1971, p. 167.

4. Williams, p. 171.

5. Smith, p. 183.

6. Williams, p. 172.

7. Williams, p. 171.

8. Williams, p. 175.

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Appendix I

A Method for Determining Resonant Stabilities

One of the interesting side results of this investigation has been the discovery that certain resonances are more stable against perturbations in the oscillations than others. This also has a definite bearing on the total number of resonant combinations that can exist. We defined a resonance as any system where the periods of the bodies were the ratio of two small whole numbers. Taking this to be any number less than ten and eliminating multiples and so-called mirror resonances; that is, a 3/1 resonance is the same as a 1/3 resonance except viewed from the perturbing planet instead of from the perturbed planet as usual, we are still left with 27 different combinations.

While each resonance is stable in its non-oscillating form; if it is perturbed or is oscillating a majority of these patterns get perturbed into another orbit. A possible explanation for this can be found by going back to the basic mechanism that maintains a resonance, the oscillations of the conjunction point. These oscillations show up as small variations in the relative period of the outer planet relative to the inner. For a 3/2 resonance the period between perihelion passages is 1.50 orbits. Similarly, for a 4/3 the period is 1.33 and for an 8/7 the period is 1.14, or in general the period is just the numerical value of the resonance ratio. The oscillations slow or speed up these periods; typically for a 3/2 resonance the values range from 1.47 to 1.53. Right away we see that a resonance must have a certain amount of "room" to oscillate in for it to be distinguishable as an oscillating resonance. If there was another resonance within the range of the oscillations there

would be some sort of interference between the two.

This is the situation for an 8/7 resonance. It is stable in the non-oscillating case but if it oscillates it interference with the 9/8 at 1.13 on the low side and the 7/6 at 1.17 on the high side. In addition, the 8/7 resonance has a loop that falls near the perturbing planet so that any movement brings this loop and thus the planet closer to the perturber and consequently increases the disruptive force of the planet. The amount of relative "room" between resonances can be seen if we plot the relative periods on a number line and then look for gaps in which a resonance could oscillate. As it turns out the gaps occur around the resonances 3/2, 5/3, 4/3, 2/1, 3/1, 5/2, and slightly less stable ones around 5/4, 7/3, and 8/3 resonances. These are the same resonances that we see throughout the solar system. Virtually none of the other resonances are found.

Looking for the relative amount of "room" around a resonance is only a convenient way of finding resonances. There is more likely a basic explanation of the resonances but is interesting that this method works at all.

Appendix II

Computer Program - NEPTUNE

The following computer program is a model of the Neptune satellite system while it is orbiting the Sun. It was used to study the initial escape of Pluto from orbit around Neptune. The program is made up of four parts; a force function definition section, an initialization of variables section, an integration section using a Runge-Kutta numerical integration scheme, and finally, an output section.

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NEPTUNE

```
20REM THE FOLLOWING STATEMENTS SET UP OUTPUT FILES
21FILE #1:"YVX" 'Y VS X
22FILE #2: "XVT" 'X VS T
23 FILE#3:"3D"
25SCRATCH #1
26SCRATCH #2
27 SCRATCH#3
99'
100REM THE FOLLOWING STATEMENTS ARE ACCELERATIONS
105LET P1=12.5663706144
110 LET P2=39.4784176044
120DEF FNA(A,B,C,D,E)
121LET FNA= -764966.42*(A+11983.755)/(((A+11983.755)^2+B^2)^(1.5))
122 LET FNA=FNA-P2*A*(A*A+B*B)^(-1.5)+4.4533773E-7*(A+11983.755)
123 LET FNA=FNA+.001334673*C
124
129FNEND
130DEF FNB(A,B,C,D,E)
131 LET FNB=-764966.42*B*((A+11983.755)^2+B*B)^(-1.5)-P2*B*(A*A+B*B)^(-
132 LET FNB=FNB+4.4533773E-7*B-.001334673*C
139FNEND
140 DEF FNC(A,B,C,D,E)
141 LET FNC=-P2*E*(A^2+B^2+E^2)^(-1.5)
142 IF M=0 THEN 149
143 LET Q1=A-1-(1-D)*(COS(6.283185307*T0)-1)
144 LET Q2=B-(1-D)*SIN(6.283185307*T0)
146 LET FNC=FNC-P2*M*E*(Q1^2+Q2^2+E^2)^(-1.5)
149 FNEND
199'
200REM THE FOLLOWING ARE INITIALIZING STEPS
201PRINT"INPUT RATIO OF PERTURBING TO PRIMARY MASS. IF THERE IS NO"
202PRINT"PERTURBING MASS, INPUT 0"
203INPUT M
206PRINT"INPUT 0,1 FOR NON-ROTATING OR ROTATING COORDINATES"
207INPUT D
209PRINT"INPUT INITIAL X, VX"
210INPUT X,U
215PRINT"INPUT INITIAL Y,VY"
220INPUT Y,V
222 PRINT "INPUT INITIAL Z,VZ"
223 INPUT Z,W
225PRINT"INPUT STARTING TIME, END TIME, # STEPS,OUTPUTSTEP INTERVAL"
230INPUT T0,T1,N,N1
235LET H=(T1-T0)/N
245 LET R=SQR(X^2+Y^2+Z^2)
246 LET T2=(180*ATN(Y/X))/3.14159
247 IF X>0 THEN 250
248 LET T2=T2+180
250 PRINT #3:X,",",Y,",",Z
255PRINT #2: T0,X
```

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NEPTUNE (continued)

```
399'
400REM THE FOLLOWING STEPS PERFORM THE INTEGRATION
401 FOR I=1 TO N
406REM K1 GOES WITH U, K2 WITH V, K3 WITH X, K4 WITH Y,K5 WITH W,K6WITH Z
410LET K1=H*FNA(X,Y,V,D,Z)
415LET K2=H*FNB(X,Y,U,D,Z)
417 LET K5=H*FNC(X,Y,W,D,Z)
420LET K3=H*U
425LET K4=H*V
427 LET K6=H*W
430LET U1=U+K1/6
435LET V1=V+K2/6
437 LET W1=W+K5/6
440LET X1=X+K3/6
445LET Y1=Y+K4/6
447 LET Z1=Z+K6/6
450LET U2=U+K1/2
455LET V2=V+K2/2
457 LET W2=W+K5/2
460LET X2=X+K3/2
465LET Y2=Y+K4/2
467 LET Z2=Z+K6/2
470LET K1=H*FNA(X2,Y2,V2,D,Z2)
475LET K2=H*FNB(X2,Y2,U2,D,Z2)
477 LET K5=H*FNC(X2,Y2,W2,D,Z2)
480LET K3=H*U2
485LET K4=H*V2
487 LET K6=H*W2
490LET U1=U1+K1/3
495LET V1=V1+K2/3
497 LET W1=W1+K5/3
500LET X1=X1+K3/3
505LET Y1=Y1+K4/3
507 LET Z1=Z1+K6/3
510LET U2=U+K1/2
515LET V2=V+K2/2
517 LET W2=W+K5/2
520LET X2=X+K3/2
525LET Y2=Y+K4/2
527 LET Z2=Z+K6/2
530LET K1=H*FNA(X2,Y2,V2,D,Z2)
535LETK2=H*FNB(X2,Y2,U2,D,Z2)
537 LET K5=H*FNC(X2,Y2,W2,D,Z2)
540LET K3=H*U2
545LET K4=H*V2
547 LET L6=H*W2
550LET U1=U1+K1/3
555LET V1=V1+K2/3
557 LET W1=W1+K5/3
560LET X1=X1+K3/3
```

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NEPTUNE (continued)

```
565LET Y1=Y1+K4/3
567 LET Z1=Z1+K6/3
570LET U2=U+K1
575LET V2=V+K2
577 LET W2=W+K5
580LET X2=X+K3
585LET Y2=Y+K4
587 LET Z2=Z+K6
590LET K1=H*FNA(X2,Y2,V2,D,Z2)
595LET K2=H*FNB(X2,Y2,U2,D,Z2)
597 LET K5=H*FNC(X2,Y2,W2,D,Z2)
600LET K3=H*U2
605LET K4=H*V2
607 LET K6=H*W2
610LET U=U1+K1/6
615LET V=V1+K2/6
617 LET W=W1+K5/6
620LET X=X1+K3/6
625LET Y=Y1+K4/6
627 LET Z=Z1+K6/6
630LET T0=T0+H
635IF INT(I/N1)-(I/N1)<>0 THEN 665
640 LET R=SQR(X^2+Y^2+Z^2)
641 LET T2=(180*ATN(Y/X))/3.14159
642 IF X>0 THEN 645
643 LET T2=T2+180
645PRINT #1:X,Y
650PRINT #2:T0,X
665 PRINT #3:X;",";Y;",";Z
670 NEXT I
1140PRINT"FINAL TIME = ";T0
1145PRINT"FINAL X= ";X;"VX= ";U
1147 PRINT"FINAL Y=";Y;"VY=";V
1150 PRINT "FINAL Z=";Z;" VZ=";W
8000RESET #1
8010RESET #2
8030 RESET #3
9000PRINT
9999END
```

Appendix III

Computer Program - LONGZUSJ

The following program is the complete model of the outer solar system containing all the plants from Jupiter outwards. The program can be run in both a Cartesian fixed coordinate frame or in a rotating relative coordinate frame. The name LONGZUSJ is derived from the fact that the program was designed to study the 3/2 Pluto - Neptune resonance over long time scales and while doing so would print out the z-position and z-velocity of Pluto each time Pluto was at perihelion so that we could study the precession of the perihelion. In addition, the program contains the planets Uranus, Saturn, and Jupiter. Each planet is defined by its position in space in each of the three coordinates using only three lines in the program. For example, Saturn's position is computed by lines 293-294, 393-394, and 493-494. Saturn's perturbation on Pluto is then computed and processed at each step of the Runge-Kutta integration sequence with is almost identical to that in the program NEPTUNE.

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LONGZUSJ

```
1 DIM P(100)
2 SETDIGITS 7
20 LET P7=35.14857947
100REM THE FOLLOWING STATEMENTS SET UP OUTPUT FILES
110 FILE #1:"INITIAL2"
120 FILE #2:"DATA3"
130 FILE #3:"XVX1"
135FILE#4:"XY3"
140'
150REM THE FOLLOWING STATEMENTS ARE ACCELERATIONS
160 LET P3=.63786
170 LET P4=12.32461
180 LET P5=4.3606E-5
185 LET P9=.1729238
190LET P1=12.5663706144
195 LET P6=.314895887
200 LET P2=39.4784176044
202 LET P(10)=87.28965
205 LET P8=2.856E-4
207 LET P(11)=9.547966E-4
210DEF FNA(A,B,C,D,E)
220LET FNA=P2*(D-(A^2+B^2+E^2)^(-1.5))*A+P1*C*D
230IF M=0 THEN 300
240LET Q1=A-1-(1-D)*(COS(6.283185307*T0)-1)
250LET Q2=B-(1-D)*SIN(6.283185307*T0)
260LET FNA=FNA-P2*Q1*M*(Q1^2+Q2^2+E^2)^(-1.5)
270 LET Q3=A-P3*(COS((P4-D*6.283185307)*T0))
280 LET Q4=B-P3*SIN((P4-D*6.283185307)*T0)
290 LET FNA=FNA-P2*Q3*P5*(Q3^2+Q4^2+E^2)^(-1.5)
293 LET Q5=A-P6*(COS((P7-D*6.283185307)*T0))
294 LET Q6=B-P6*SIN((P7-D*6.283185307)*T0)
295 LET FNA=FNA-P2*Q5*P8*(Q5^2+Q6^2+E^2)^(-1.5)
296 LET Q7=A-P9*(COS((P(10)-D*6.283185307)*T0-123.2285))
297 LET Q8=B-P9*SIN((P(10)-D*6.283185307)*T0 -123.2285)
298 LET FNA=FNA-P2*Q7*P(11)*(Q7^2+Q8^2+E^2)^(-1.5)
300FNEND
310DEF FNB(A,B,C,D,E)
320LET FNB=P2*(D-(A^2+B^2+E^2)^(-1.5))*B-P1*C*D
330IF M=0 THEN 400
340LET Q1=A-1-(1-D)*(COS(6.283185307*T0)-1)
350LET Q2=B-(1-D)*SIN(6.283185307*T0)
360LET FNB=FNB-P2*Q2*M*(Q1^2+Q2^2+E^2)^(-1.5)
370 LET Q3=A-P3*(COS((P4-D*6.283185307)*T0))
380 LET Q4=B-P3*SIN((P4-D*6.283185307)*T0)
390 LET FNB=FNB-P2*Q4*P5*(Q3^2+Q4^2+E^2)^(-1.5)
393 LET Q5=A-P6*(COS((P7-D*6.283185307)*T0))
394 LET Q6=B-P6*SIN((P7-D*6.283185307)*T0)
395LET FNB=FNB-P2*Q6*P8*(Q5^2+Q6^2+E^2)^(-1.5)
396 LET Q7=A-P9*(COS((P(10)-D*6.283185307)*T0-123.2285))
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```

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```
398 LET FNB=FNB-P2*Q8*P(11)*(Q7^2+Q8^2+E^2)^(-1.5)
400FNEND
410 DEF FNC(A,B,C,D,E)
420 LET FNC=-P2*E*(A^2+B^2+E^2)^(-1.5)
430 IF M=0 THEN 500
440 LET Q1=A-1-(1-D)*(COS(6.283185307*T0)-1)
450 LET Q2=B-(1-D)*SIN(6.283185307*T0)
460 LET FNC=FNC-P2*M*E*(Q1^2+Q2^2+E^2)^(-1.5)
470 LET Q3=A-P3*(COS((P4-D*6.283185307)*T0))
480 LET Q4=B-P3*SIN((P4-D*6.283185307)*T0)
490 LET FNC=FNC-P2*E*P5*(Q3^2+Q4^2+E^2)^(-1.5)
493 LET Q5=A-P6*(COS((P7-D*6.283185307)*T0))
494 LET Q6=B-P6*SIN((P7-D*6.283185307)*T0)
495 LET FNC=FNC-P2*E*P8*(Q5^2+Q6^2+E^2)^(-1.5)
496 LET Q7=A-P9*(COS((P(10)-D*6.283185307)*T0-123.2285))
497 LET Q8=B-P9*SIN((P(10)-D*6.283185307)*T0 -123.2285)
498 LET FNC=FNC-P2*E*P(11)*(Q7^2+Q8^2+E^2)^(-1.5)
500 FNEND
510'
520REM THE FOLLOWING ARE INITIALIZING STEPS
530PRINT"INPUT RATIO OF PERTURBING TO PRIMARY MASS. IF THERE IS NO"
540PRINT"PERTURBING MASS, INPUT 0"
550INPUT M
560PRINT"INPUT 0,1 FOR NON-ROTATING OR ROTATING COORDINATES"
570INPUT D
580PRINT"INPUT INITIAL X, VX"
590INPUT X,U
600PRINT"INPUT INITIAL Y,VY"
610INPUT Y,V
620 PRINT "INPUT INITIAL Z,VZ"
630 INPUT Z,W
640PRINT"INPUT STARTING TIME, END TIME, # STEPS,OUTPUTSTEP INTERVAL"
650INPUT T0,T1,N,N1
660LET H=(T1-T0)/N
670 PRINT#2:X,U
680 PRINT#2:Y,V
690 PRINT#2:Z,W
700'
710REM THE FOLLOWING STEPS PERFORM THE INTEGRATION
720 FOR I=1 TO N
730REM K1 GOES WITH U, K2 WITH V, K3 WITH X, K4 WITH Y,K5 WITH W,K6WITH Z
740LET K1=H*FNA(X,Y,V,D,Z)
750LET K2=H*FNB(X,Y,U,D,Z)
760 LET K5=H*FNC(X,Y,W,D,Z)
770LET K3=H*U
780LET K4=H*V
790 LET K6=H*W
800LET U1=U+K1/6
810LET V1=V+K2/6
820 LET W1=W+K5/6
```

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LONGZUBJ (continued)

```
830LET X1=X+K3/6
840LET Y1=Y+K4/6
850 LET Z1=Z+K6/6
860LET U2=U+K1/2
870LET V2=V+K2/2
880 LET W2=W+K5/2
890LET X2=X+K3/2
900LET Y2=Y+K4/2
910 LET Z2=Z+K6/2
920LET K1=H*FNA(X2,Y2,V2,D,Z2)
930LET K2=H*FNB(X2,Y2,U2,D,Z2)
940 LET K5=H*FNC(X2,Y2,W2,D,Z2)
950LET K3=H*U2
960LET K4=H*V2
970 LET K6=H*W2
980LET U1=U1+K1/3
990LET V1=V1+K2/3
1000 LET W1=W1+K5/3
1010LET X1=X1+K3/3
1020LET Y1=Y1+K4/3
1030 LET Z1=Z1+K6/3
1040LET U2=U+K1/2
1050LET V2=V+K2/2
1060 LET W2=W+K5/2
1070LET X2=X+K3/2
1080LET Y2=Y+K4/2
1090 LET Z2=Z+K6/2
1100LET K1=H*FNA(X2,Y2,V2,D,Z2)
1110LETK2=H*FNB(X2,Y2,U2,D,Z2)
1120 LET K5=H*FNC(X2,Y2,W2,D,Z2)
1130LET K3=H*U2
1140LET K4=H*V2
1150 LET L6=H*W2
1160LET U1=U1+K1/3
1170LET V1=V1+K2/3
1180 LET W1=W1+K5/3
1190LET X1=X1+K3/3
1200LET Y1=Y1+K4/3
1210 LET Z1=Z1+K6/3
1220LET U2=U+K1
1230LET V2=V+K2
1240 LET W2=W+K5
1250LET X2=X+K3
1260LET Y2=Y+K4
1270 LET Z2=Z+K6
1280LET K1=H*FNA(X2,Y2,V2,D,Z2)
1290LET K2=H*FNB(X2,Y2,U2,D,Z2)
1300 LET K5=H*FNC(X2,Y2,W2,D,Z2)
1310LET K3=H*U2
1320LET K4=H*V2
```

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LONGZUSJ (continued)

```
1330 LET K6=H*W2
1340LET U=U1+K1/6
1350LET V=V1+K2/6
1360 LET W=W1+K5/6
1370LET X=X1+K3/6
1380LET Y=Y1+K4/6
1390 LET Z=Z1+K6/6
1400LET T0=T0+H
1410 LET R=SQR(X^2+Y^2+Z^2)
1420 LET T2=(180*ATN(Y/X))/3.14159
1429 IF INT(I/N1)-(I/N1)=0 THEN 1432
1430 GO TO 1440
1431 LET T2=T2+180
1432 PRINT#4:X;",";Y
1440 IF SQR(X*X+Y*Y)>1 THEN 1460
1441 LET T4=T0
1442 LET Z4=Z
1443 LET W4=W
1444 LET K=SQR(X*X+Y*Y)
1445 IF T4>T5+.03 THEN 1452
1446 IF K<L THEN 1456
1447 LET D2=D2+1
1448 IF D2>=2 THEN 1460
1449 PRINT#2:T5;"  ";L;"   ";Z5;"  ";W5
1450 LET T5=T5+1.4
1451 GO TO 1460
1452 LET L=1
1453 LET D2=0
1454 LET T5=T4
1455 GO TO 1460
1456 LET L=K
1457 LET T5=T4
1458 LET Z5=Z4
1459 LET W5=W4
1460 NEXT I
1470PRINT#2:"FINAL TIME = ";T0
1480 PRINT#3:T0
1490 PRINT #3:X,U
1500PRINT#2:"FINAL X= ";X;"VX= ";U
1510 PRINT#3:Y,V
1520 PRINT#2:"FINAL Y=";Y;"VY=";V
1530 PRINT #2:"FINAL Z=";Z;" VZ=";W
1540 PRINT #3:Z,W
1550PRINT
1560END
```

Appendix IV

Computer Program - CONV2

The program CONV2 was used to convert the positions and velocities of Pluto in each of the three coordinates that were obtained in the Neptune frame (See pragram NEPTUNE in Appendix II) into positions and velocities in the Sun center solar system. These initial conditions were given in both the rotating and non-rotating coordinate systems.

CONV2

```
5 A=0
10 PRINT "INPUT X,VX,Y,VY,T"
20 INPUT X,V1,Y,V2,T
30 LET O=T*(6.6733629E-4)
35 LET Y=Y*3.755E8/4.4999E12
40 LET X1=(X*3.755E8/4.4999E12+1)*COS(O)-Y*SIN(O)
50 LET Y1=(X*3.755E8/4.4999E12+1)*SIN(O)+Y*COS(O)
60 LET V1=V1*.786455782- 6.283185308*SIN(O)
70 LET V2=V2*.786455782+6.283185308*COS(O)
71 LET V3=V1*COS(O)-V2*SIN(O)
72 LET V4=V1*SIN(O)+V2*COS(O)
73 LET X=X1
75 LET Y=Y1
76 LET V1=V3
77 V2=V4
90 B=.299673033
100 Y=Y*COS(B)
110 V2=V2*COS(B)
120 Z=Y*SIN(B)
130 V3=V2*SIN(B)
191 PRINT "NON-ROTATING"
192 PRINT "X=";X;"VX=";V1
193 PRINT "Y=";Y;"VY=";V2
194 PRINT "Z=";Z;"VZ=",V3
195 LET A=A+1
196 IF A=2 THEN 999
200 V1=V1+6.283185308*Y
210 V2=V2-6.283185308*X
215 PRINT "ROTATING"
216 GO TO 192
999 END
```

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REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER U.S.N.A. - TSPR.; no. 91 (1978)	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) (6) INITIAL CONDITIONS FOR AN ORBITAL RESONANCE IN A SATELLITE SYSTEM.		5. TYPE OF REPORT & PERIOD COVERED Final, 1977/78.
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) (10) Stephen M. Hopkins		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS United States Naval Academy, Annapolis, Md.		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS United States Naval Academy, Annapolis, md.		12. REPORT DATE (11) 1 June 1978
		13. NUMBER OF PAGES 76 leaves.
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office) (14) USNA-TSPR-91		15. SECURITY CLASS. (of this report) UNCLASSIFIED.
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

(9) Final rept. 1977-1978,

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17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

This document has been approved for public release; its distribution is UNLIMITED. (12) 80p.

18. SUPPLEMENTARY NOTES

Accepted by the chairman of the Trident Scholar Committee.

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Pluto (Planet)
Neptune (Planet)
Orbital motion
Frequencies of oscillating system.

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This project undertook to examine the conditions of the 3/2 resonance between the planets of Pluto and Neptune and try to determine whether was it possible for Pluto to have escaped from Neptune and became his moon.

The writer uses numerical integration and computer programs to establish that the Neptune and Pluto are in resonance, consequently they would never come close enough to one another for Pluto to ~~have been~~ be a moon.

In conclusion: the writer has eliminated the major objection to the moon hypothesis as to Pluto's origin.

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